The Ingredients of a Typical Mass Flux Scheme

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Outline for today

- Recap / starting point
- Justifications for “bulk” schemes
- Main ingredients of a typical bulk scheme:
  - Vertical structure of convection
  - Overall amount of convection

Recap: Starting Point

- Interactions of convection and large-scale dynamics crucial
- Need for a convective parameterization in GCMs and (most) NWP

  Assume we are thinking of a parent model with grid length 20 to 100km
- Basic idea: represent effects of a set of hot towers / plumes / convective clouds within the grid box

$\Rightarrow$

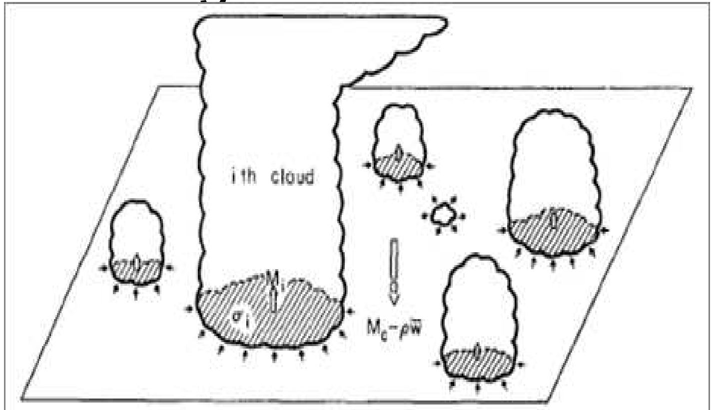


FIG. 1. A unit horizontal area at some level between cloud base and the highest cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud which has lost buoyancy is shown detraining cloud air into the environment.

Recap: Starting Point

- Assume that there exists a meaningful “large-scale” within which the convective systems are embedded

  this is a little suspect

- Assume that the “large-scale” is well described by the grid box state in the parent model

  this is more suspect

- Aim of the parameterization is to determine the tendencies of grid-box variables due to convection, given the grid-box state as input

Recap: Starting Point

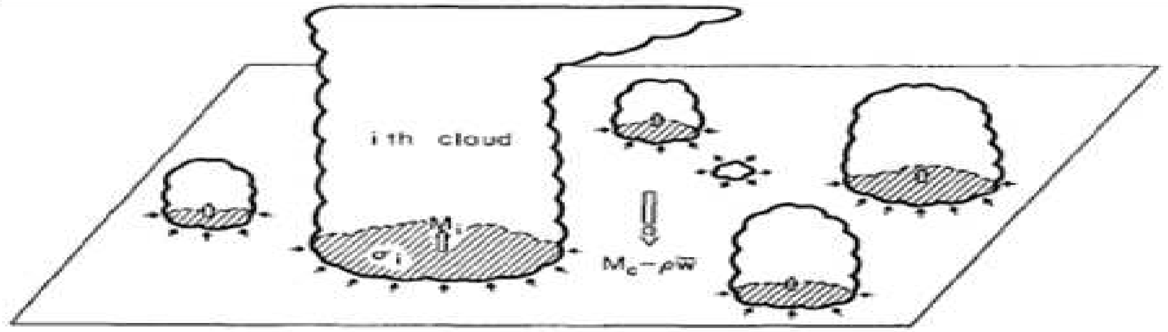


FIG. 1. A unit horizontal area at some level between cloud base and the highest cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud which has lost buoyancy is shown detraining cloud air into the environment.

- Convection characterised by ensemble of non-interacting convective plumes within some area of tolerably uniform forcing
- Individual plume equations formulated in terms of mass flux, $M_i = \rho\sigma_i w_i$

Mass flux approximation

- Effects of the plumes on their environment are very simple under the usual mass flux approximations of $w \ll w_i$ and $\sigma_i \ll 1$.
- For some intensive variable χ

$$\rho\overline{\chi' w'} \approx \sum_i M_i(\chi_i - \chi)$$

where the prime is a local deviation from the horizontal mean

- Recall that the vertical derivative of this provides a tendency for the large-scale flow

Basic questions

Supposing we accept all the above, we still need to ask...

1. How should we formulate the entrainment and detrainment?
   ie, what is the vertical structure of the convection?
2. How should we formulate the closure?
   ie, what is the amplitude of the convective activity?
3. Do we really need to make calculations for every individual plume in the grid box?
   ie, is our parameterization practical and efficient?

We consider 3 first, because the answer has implications for 1 and 2.

Do we really need to make calculations for every individual plume in the grid box?

Basic idea of spectral method

- Group the plumes together into types defined by a labelling parameter λ
- In Arakawa and Schubert (1974) this is the fractional entrainment rate, $\lambda = E/M$, but it could be anything
- e.g. cloud top height $\lambda = z_T$ is sometimes used
- a generalization to multiple spectral parameters would be trivial

Basic idea of bulk method

- Sum over plumes and approximate ensemble with a representative “bulk” plume
- This can only be reasonable if the plumes do not interact directly, only with their environment
- And if plume equations are almost linear in mass flux
- Summation over plumes will recover equations with the same form so the sum can be represented as a single equivalent plume

Mass-flux weighting

We will use the mass-flux-weighting operation (Yanai et al. 1973)

$$\chi_{\mathrm{bulk}} = \frac{\sum M_i \chi_i}{\sum M_i}$$

χ_{bulk} is the bulk value of χ produced from an average of the χ_i for each individual plume

Plume equations

$$\frac{\partial \rho \sigma_i}{\partial t} = E_i - D_i - \frac{\partial M_i}{\partial z}$$

$$\frac{\partial \rho \sigma_i s_i}{\partial t} = E_i s - D_i s_i - \frac{\partial M_i s_i}{\partial z} + L\rho c_i + \rho Q_{Ri}$$

$$\frac{\partial \rho \sigma_i q_i}{\partial t} = E_i q - D_i q_i - \frac{\partial M_i q_i}{\partial z} - \rho q_i$$

$$\frac{\partial \rho \sigma_i l_i}{\partial t} = -D_i l_i - \frac{\partial M_i l_i}{\partial z} + \rho c_i - R_i$$

- $s = c_p T + gz$ is the dry static energy
- Q_R is the radiative heating rate
- R is the rate of conversion of liquid water to precipitation
- c is the rate of condensation

Using the plume equations

Average over the plume lifetime to get rid of $\partial/\partial t$:

$$E_i - D_i - \frac{\partial M_i}{\partial z} = 0$$

$$E_i s - D_i s_i - \frac{\partial M_i s_i}{\partial z} + L\rho c_i + \rho Q_{Ri} = 0$$

$$E_i q - D_i q_i - \frac{\partial M_i q_i}{\partial z} - \rho q_i = 0$$

$$D_i l_i + \frac{\partial M_i l_i}{\partial z} + \rho c_i + R_i = 0$$

Integrate from cloud base z_B up to terminating level z_T where the in-cloud buoyancy vanishes

Neutral buoyancy level

- Occurs when the in-plume virtual temperature equals that of the environment
- Applying this and the saturation condition, the values of the detraining variables are

$$l_i = \widehat{l}$$

$$s_i = \widehat{s} = s - \frac{L\varepsilon}{1+\gamma\varepsilon\delta}\left(\delta(q^* - q) - \widehat{l}\right)$$

$$q_i = \widehat{q}* = q^* - \frac{\gamma\varepsilon}{1+\gamma\varepsilon\delta}\left(\delta(q^* - q) - \widehat{l}\right)$$

where

$$\varepsilon = \frac{c_p T}{L} \quad ; \quad \delta = 0.608 \quad ; \quad \gamma = \frac{L}{c_p} \left.\frac{\partial q^*}{\partial T}\right|_p$$

Effects on the environment

Taking a mass-flux weighted average,

$$\rho\overline{\chi' w'} \approx \sum_i M_i(\chi_i - \chi) = M(\chi_{\text{bulk}} - \chi)$$

where

$$M = \sum_i M_i$$

Recall that the aim is for the equations to take the same form as the individual plume equations but now using bulk variables like M and χ_{bulk}

Equivalent bulk plume I

Now look at the weighted-averaged plume equations

$$E - D - \frac{\partial M}{\partial z} = 0$$

$$Es - \sum_i D_i s_i - \frac{\partial M s_{\text{bulk}}}{\partial z} + L\rho c + \rho QR = 0$$

$$Eq - \sum_i D_i q_i - \frac{\partial M q_{\text{bulk}}}{\partial z} - \rho c = 0$$

$$-\sum_i D_i l_i - \frac{\partial M l_{\text{bulk}}}{\partial z} + \rho c - R = 0$$

The same bulk variables feature here

Equivalent bulk plume II

$$E - D - \frac{\partial M}{\partial z} = 0$$

$$Es - \sum_i D_i s_i - \frac{\partial M s_{\text{bulk}}}{\partial z} + L\rho c + \rho Q_R = 0$$

$$Eq - \sum_i D_i q_i - \frac{\partial M q_{\text{bulk}}}{\partial z} - \rho c = 0$$

$$- \sum_i D_i l_i - \frac{\partial M l_{\text{bulk}}}{\partial z} + \rho c - R = 0$$

where

$$E = \sum_i E_i \quad ; \quad D = \sum_i D_i$$

The entrainment dilemma

- E and D encapsulate both the entrainment/detrainment process for an individual cloud and the spectral distribution of cloud types
- Is it better to set E and D directly or to set E_i and D_i together with the distribution of types?
- To be discussed...

Equivalent bulk plume III

$$Es - \sum_i D_i s_i - \frac{\partial M s_{\text{bulk}}}{\partial z} + L\rho c + \rho Q_R = 0$$

where

$$Q_R(s_{\text{bulk}}, q_{\text{bulk}}, l_{\text{bulk}}, \ldots) = \sum_i Q_{Ri}(s_i, q_i, l_i, \ldots)$$

is something for the cloud-radiation experts to be conscious about

Equivalent bulk plume IV

$$Es - \sum_i D_i s_i - \frac{\partial M s_{\text{bulk}}}{\partial z} + L\rho c + \rho Q_R = 0$$

$$Eq - \sum_i D_i q_i - \frac{\partial M q_{\text{bulk}}}{\partial z} - \rho c = 0$$

$$-\sum_i D_i l_i - \frac{\partial M l_{\text{bulk}}}{\partial z} + \rho c - R = 0$$

where

$$c(s_{\text{bulk}}, q_{\text{bulk}}, l_{\text{bulk}}, \ldots) = \sum_i c_i(s_i, q_i, l_i, \ldots)$$

$$R(s_{\text{bulk}}, q_{\text{bulk}}, l_{\text{bulk}}, \ldots) = \sum_i R_i(s_i, q_i, l_i, \ldots)$$

is something for the microphysics experts to be conscious about

A Note on Microphysics

In Arakawa and Schubert 1974, the rain rate is

$$R_i = C_0 M_i l_i$$

where C_0 is a constant. Hence,

$$R = C_0 M l_{\text{bulk}}$$

- If C_0 were to depend on the plume type then we couldn't write R as a function of the bulk quantities but would need to know how l_{bulk} is partitioned across the spectrum
  $\Longrightarrow$ A bulk scheme is committed to crude microphysics
- But microphysics in any mass-flux parameterization has issues anyway

Equivalent bulk plume V

$$E - D - \frac{\partial M}{\partial z} = 0$$

$$Es - \sum_i D_i s_i - \frac{\partial M s_{\mathrm{bulk}}}{\partial z} + L\rho c + \rho Q_R = 0$$

$$Eq - \sum_i D_i q_i - \frac{\partial M q_{\mathrm{bulk}}}{\partial z} - \rho c = 0$$

$$-\sum_i D_i l_i - \frac{\partial M l_{\mathrm{bulk}}}{\partial z} + \rho c - R = 0$$

How can we handle these terms?

(a) Below the plume tops?

(b) At the plume tops?

(a) Below the plume tops

One option is to consider all the constitutent plumes to be entraining-only (except for the detrainment at cloud top)

- If $D_i = 0$ then $\sum_i D_i \chi_i = 0$ and the problem goes away!
- This is exactly what Arakawa and Schubert did

(a) Below the plume tops

If we retain entraining/detraining plumes then we have

$$\sum_i D_i \chi_i \equiv D_\chi \chi_{\mathrm{bulk}}$$

$$D_\chi = M \frac{\sum_i D_i \chi_i}{\sum_i M_i \chi_i}$$

- The detrainment rate is $\neq \sum_i D_i$
- i.e., it is different from the D that we see in the vertical mass flux profile equation
- and it is different for each in-plume variable

$\Longrightarrow$ A bulk parameterization can only be fully equivalent to a spectral parameterization of entraining plumes

(b) At the plume tops

- There are the contributions to $\sum_i D_i \chi_i$ from plumes the that have reach neutral buoyancy at the current level
- We can use our earlier formulae for s_i etc. coming from the neutral buoyancy condition

$$Es - D\widehat{s} - \frac{\partial M s_{\text{bulk}}}{\partial z} = 0$$

$$Eq - D\widehat{q}^{*} - \frac{\partial M q_{\text{bulk}}}{\partial z} = 0$$

$$-D\widehat{l} - \frac{\partial M l_{\text{bulk}}}{\partial z} = 0$$

so now these equations use the same D as in the mass flux profile equation. But what about $\widehat{s}, \widehat{q}, \widehat{l}$?

(b) At the plume tops

Recall:

$$l_i = \widehat{l}$$

$$s_i = \widehat{s} = s - \frac{L\varepsilon}{1+\gamma\varepsilon\delta}\left(\delta(q^* - q) - \widehat{l}\right)$$

$$q_i = \widehat{q}* = q^* - \frac{\gamma\varepsilon}{1+\gamma\varepsilon\delta}\left(\delta(q^* - q) - \widehat{l}\right)$$

- Everything on the RHS is known in the bulk system, apart from $\widehat{l}$
- $\widehat{l}(z)$ can only be calculated by integrating the plume equations for a plume that detrains at $z_i = z$

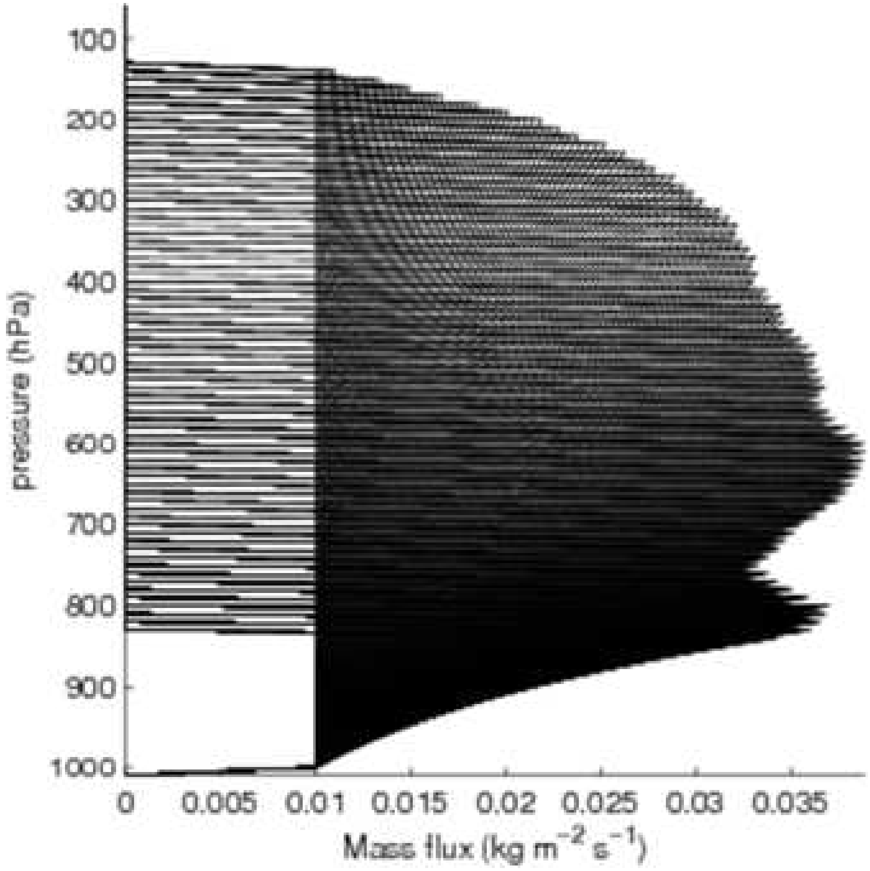
Key bulk assumption

At the heart of bulk models is an ansatz that the liquid water detrained *from each individual plume* is given by the *bulk value*

$$l_i = l_{\mathrm{bulk}}$$

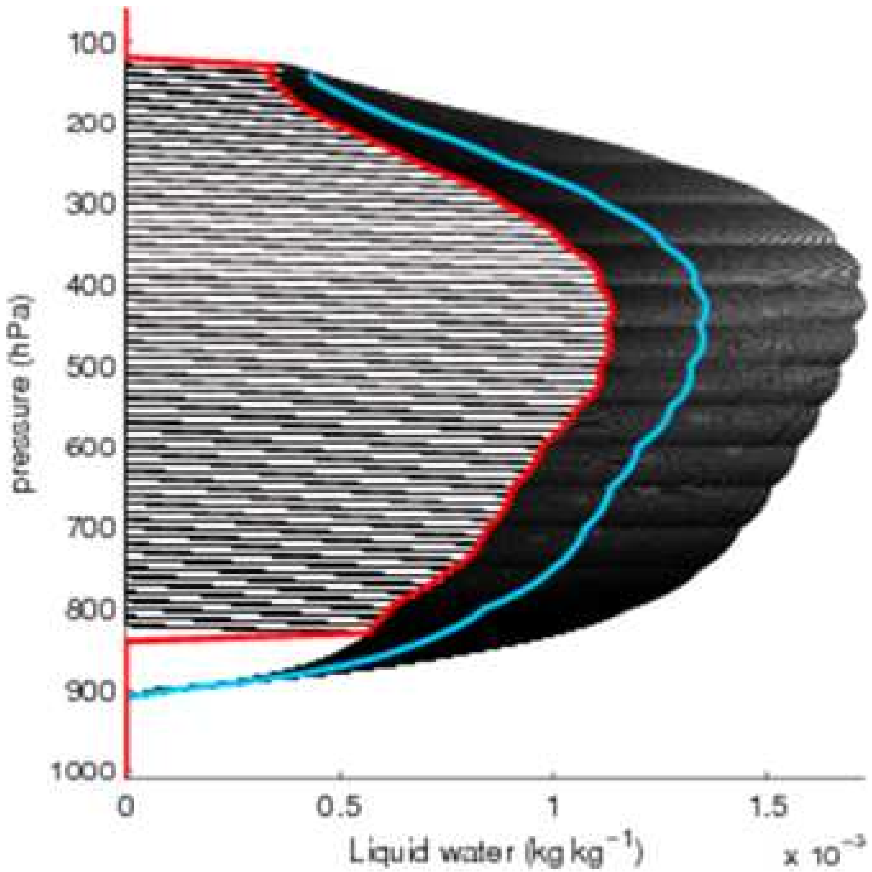
Yanai et al (1973): “gross assumption but needed to close the set of equations”

Example for Jordan sounding



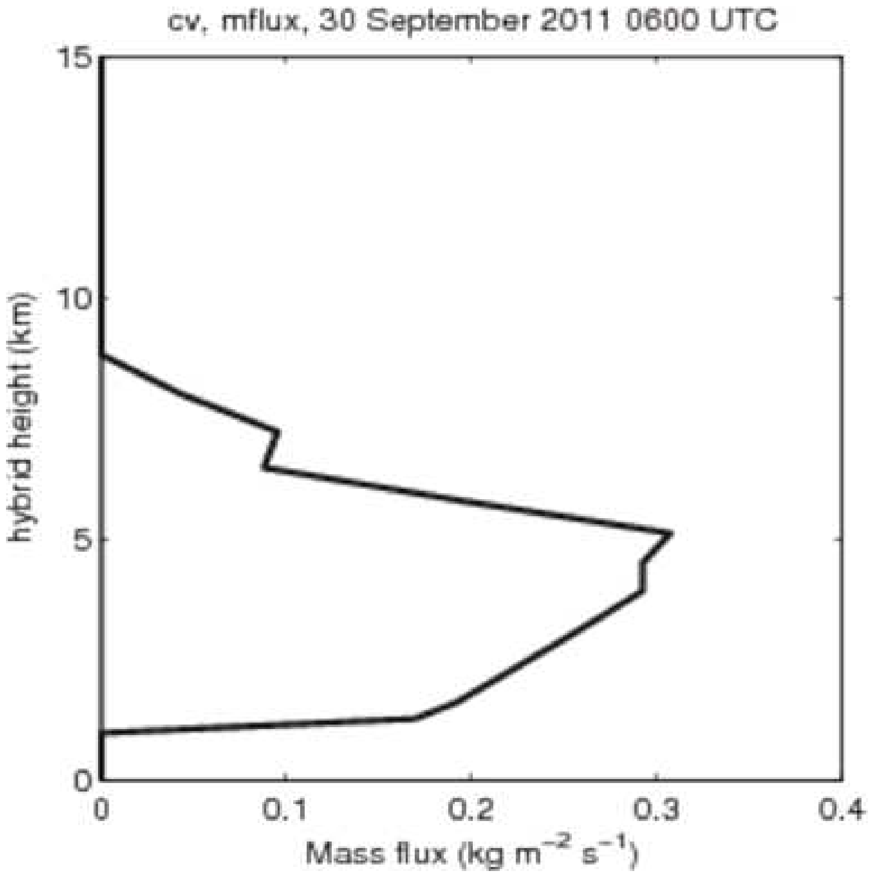
- 350 entraining plumes for typical tropical sounding
- each with an arbitrary mass flux at cloud base
- a range of entrainment rates

Detrainment Ansatz for l



- Red: detrained liquid water, $\widehat{l}$
- Blue: bulk liquid water, l_{bulk}
- liquid water is detrained throughout profile
- and always over-estimated (the detraining plumes have lower l_i)

Spectral decomposition of bulk system



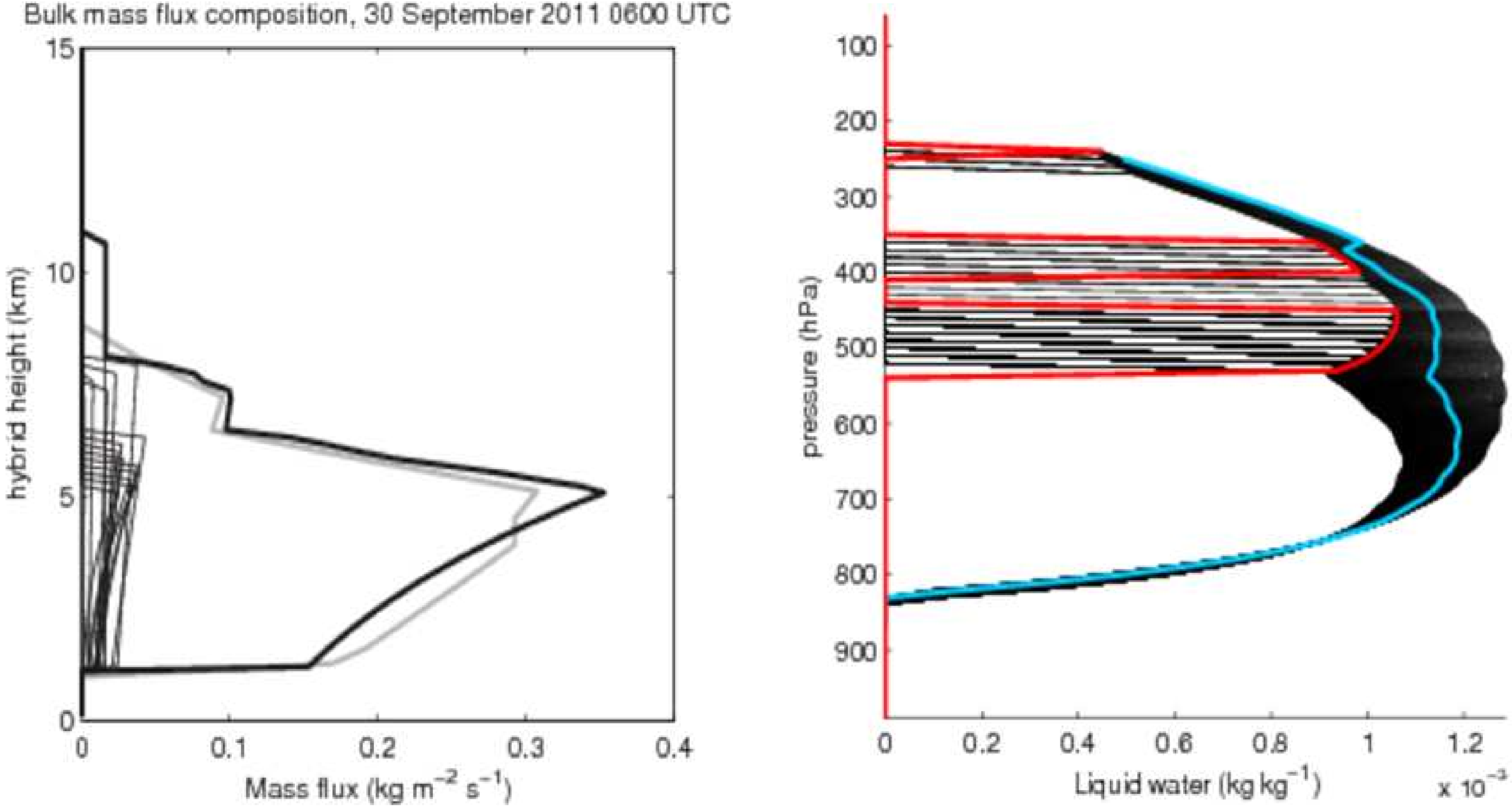
Output from UM bulk scheme of convection embedded within cold front

Construct plume ensemble using

$$\min \left| M(z) - \sum c_i M_i(z) \right| \quad c_i \geq 0$$

with M_i for entraining plumes

Spectral decomposition

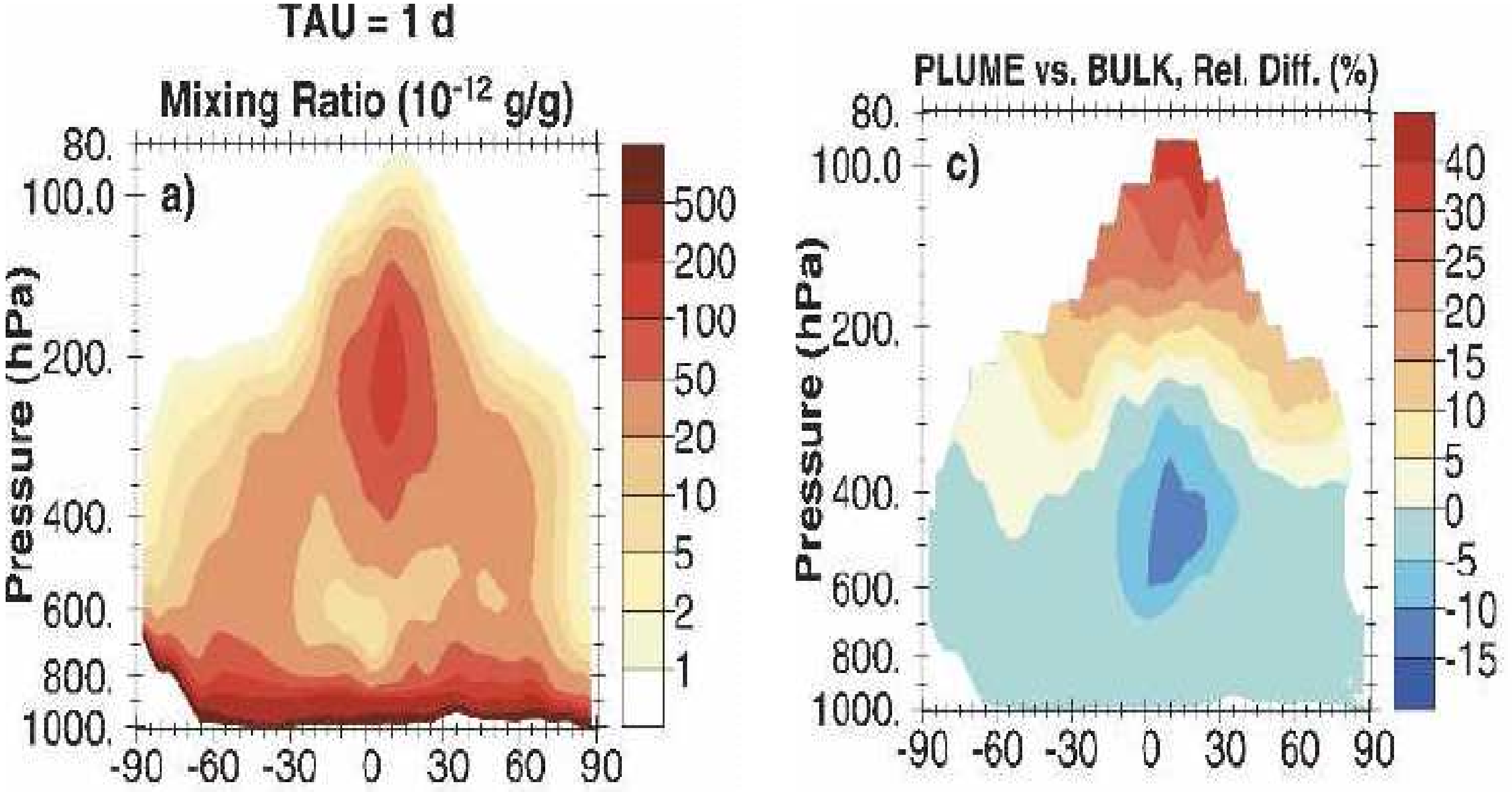


Other transports

- Contributions to $\sum_i D_i \chi_i$ from detrainment at plume top can be simplified for s, q and l from the neutral-buoyancy condition (with l ansatz)
- But no simplification occurs for other transports (e.g., tracer concentrations, momentum)
- Needs further ansatze, $\widehat{\chi}_i = \chi_{\mathrm{bulk}}$
- Or decompose bulk plume into spectrum of plumes

Example for passive scalar

Passive scalar distribution for bulk and spectral systems



From decomposition of ZM outputs (Lawrence and Rasch 2005)

Conclusions I

- A bulk model of plumes does not follow immediately from averaging over bulk plumes, but requires some extra assumptions
- Entrainment formulation is a big issue (as always!)
- In bulk systems, cloud-radiation interactions have to be estimated using bulk variables
- In bulk systems, microphysics has to be calculated using bulk variables
  - This implies very simple, linearized microphysics
  - But microphysics is problematic for mass flux methods anyway, owing to non-separation of σ_i and w_i

Conclusions II

- A bulk plume is an *entraining/detraining plume* that is equivalent to *an ensemble of entraining plumes*
- A bulk system needs a “gross assumption” that $l = l_{\mathrm{bulk}}$
  not often recognized, but relevant when detrained condensate is used as a source term for prognostic representations of stratiform cloud (for example)
- Detrained condensate from a bulk scheme is an overestimate

Bulk schemes are much more efficient, but they do have their limitations

Entrainment and detrainment

Direct estimates

- Mass continuity over a homogeneous area gives

$$\frac{\partial \sigma_c}{\partial t} + \frac{1}{A}\oint \hat{\mathbf{n}} \cdot (\mathbf{u} - \mathbf{u}_{\rm int}) dl + \frac{\partial \sigma_c w_c}{\partial z} = 0$$

- Hard to evaluate, particularly to get reliable estimates of interface velocity $\mathbf{u}_{\rm int}$
- Need to make careful subgrid interpolation (e.g., Romps 2010, Dawe and Austin 2011)
- Typically larger because
  - detraining air near cloud edge is typically less "cloud-like" than $\chi_{\rm bulk}$
  - entraining air near cloud edge is typically less "environment-like" than χ

LES diagnoses

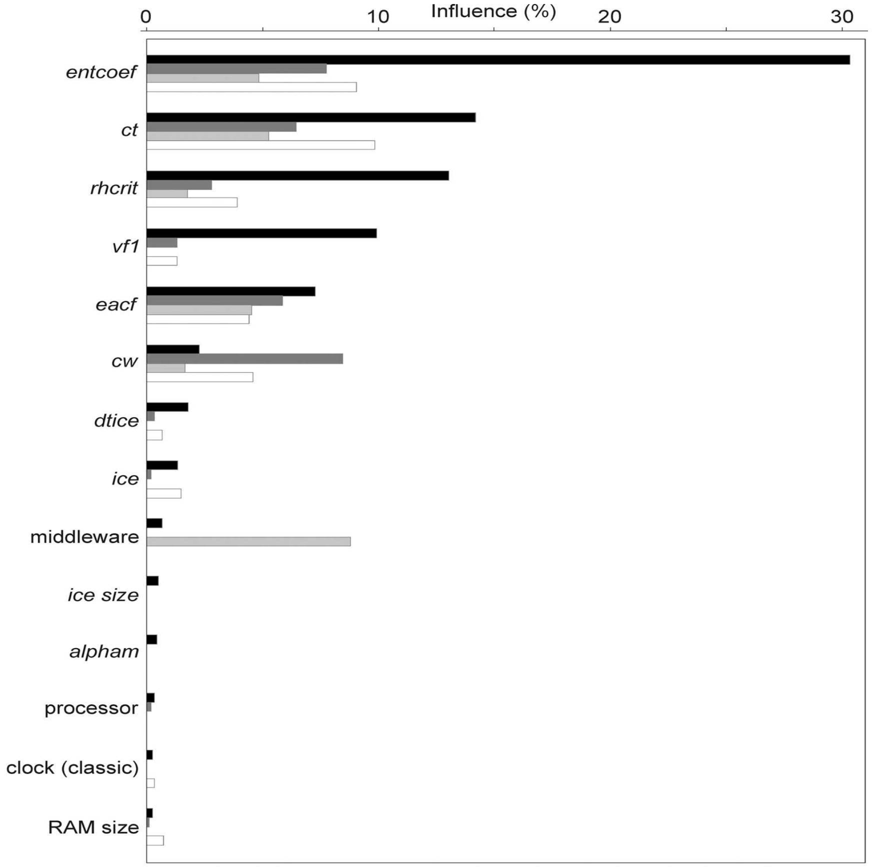
- Can make bulk estimate directly from parameterization formulae

$$\frac{1}{M}\frac{\partial M}{\partial z} = \varepsilon - \delta$$

$$\frac{\partial M\chi}{\partial z} = M(\varepsilon_\chi \chi - \delta_\chi \chi_{\mathrm{bulk}})$$

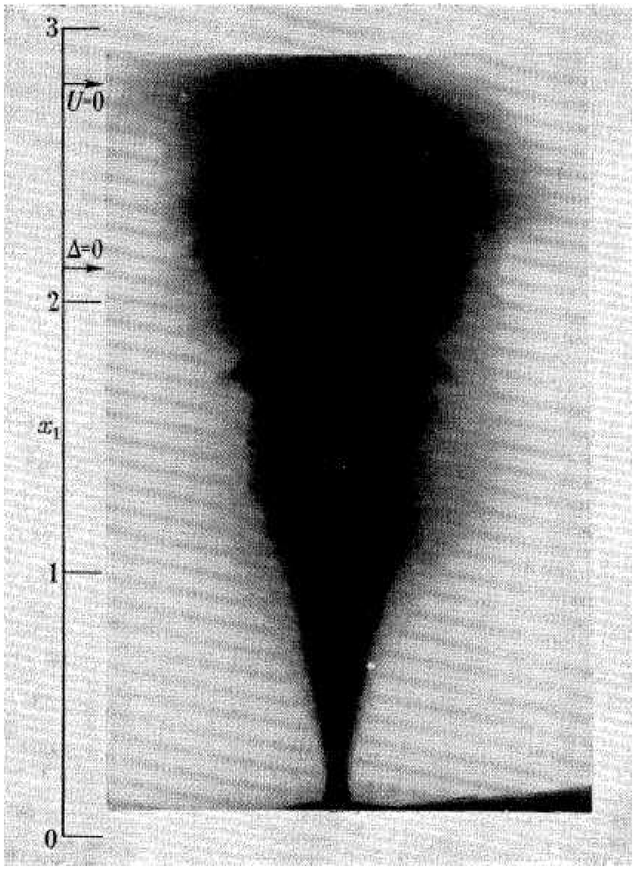
- Sampling is a key issue to define “cloud” and “environment”
  - “cloud core” $q_l > 0$, $\theta_v > \overline{\theta_v}$ often chosen

Importance of entrainment



- entrainment parameter is one of the most sensitive aspects of GCMs
- plot shows variation in climate sensitivity explained by varying different parameters in UM (Knight 2007)

Morton tank experiments



- water tank experiments (Morton et al 1956)
- growth described by fractional entrainment rate,

$$\frac{1}{M}\frac{\partial M}{\partial z} = \varepsilon \simeq \frac{0.2}{R}$$

- The form is essentially a dimensional argument
- Used for cloud models from the 1960s on

Key Issues

- lateral or cloud-top entrainment?

  i.e., diffusion-type mixing at cloud edge or a more organized flow structure dominates

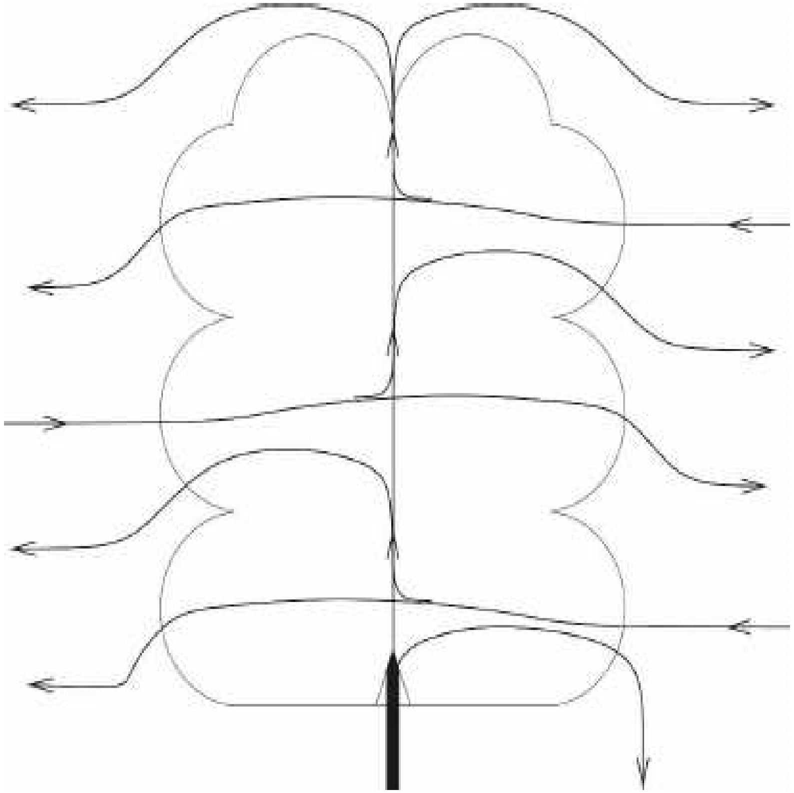
- importance of detrainment?

  unlike the lab:

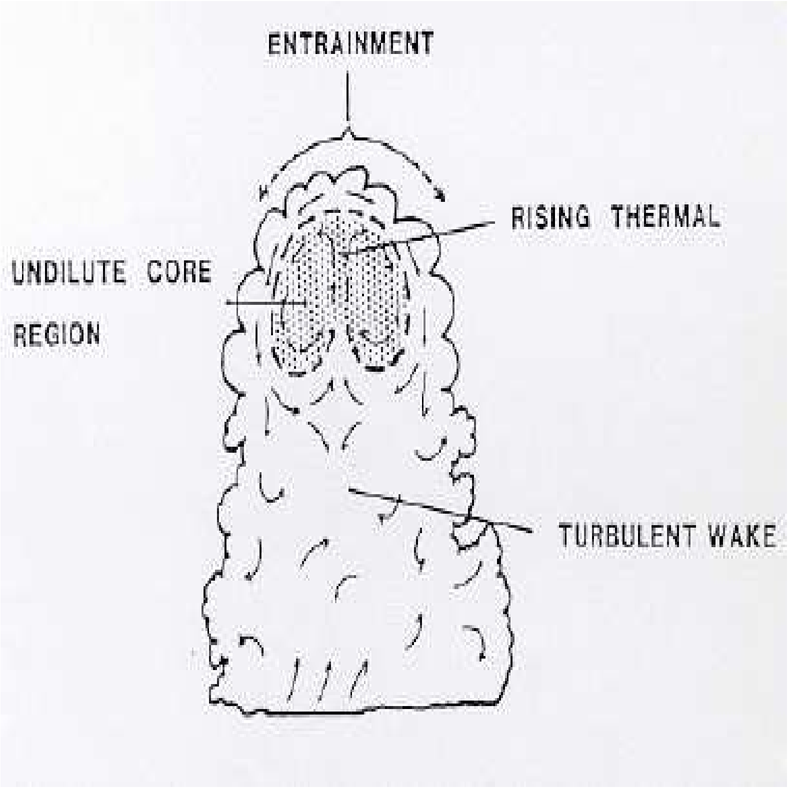
  1. turbulent mixing and evaporative cooling can cause negative buoyancy
  2. stratification means that cloud itself becomes negatively buoyant

$$\frac{1}{M}\frac{\partial M}{\partial z} = \varepsilon_{dyn} + \varepsilon_{turb} - \delta_{dyn} - \delta_{turb}.$$

Source of Entraining Air

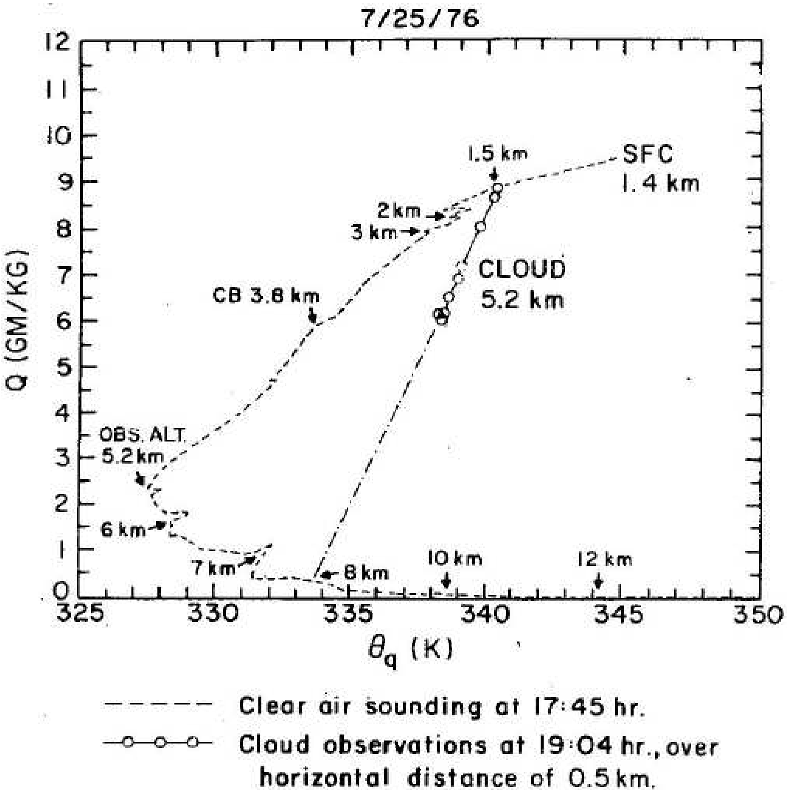


lateral entrainment
usual parameterization
assumption



cloud-top entrainment

Paluch diagrams



(Paluch 1979)

- plot conservative variables (eg, θ_e and q_T)
- in-cloud values fall along mixing line
- extrapolate to source levels: cloud-base and cloud-top
- health warning: in-cloud T is not a trivial measurement

Cloud-top entrainment

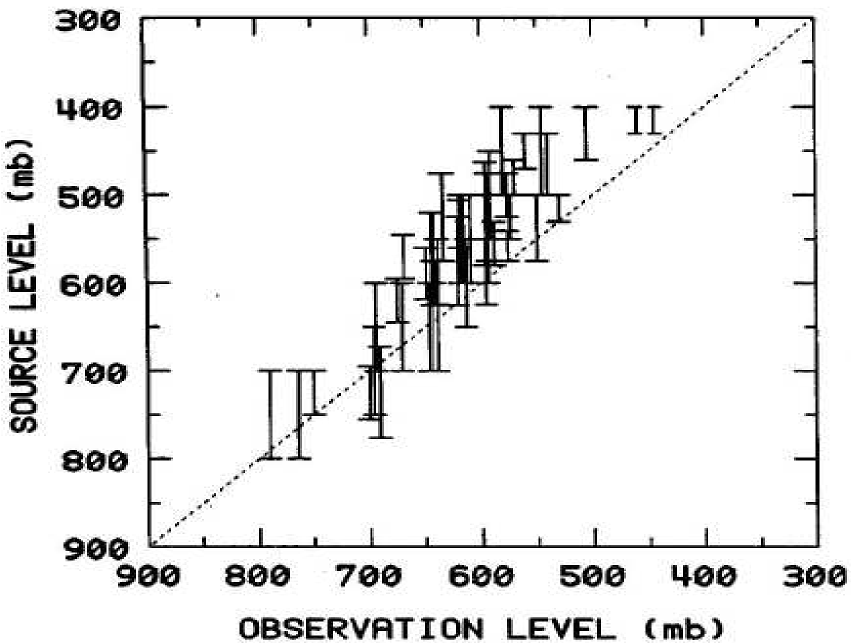


FIG. 10. The source level from which air was entrained into the cloud, as a function of the observation level in the cloud, for 44 cases taken from 44 different regions for which source levels could be determined. The error bars indicate the approximate ranges that are consistent with the observations.

(Blyth et al 1988)

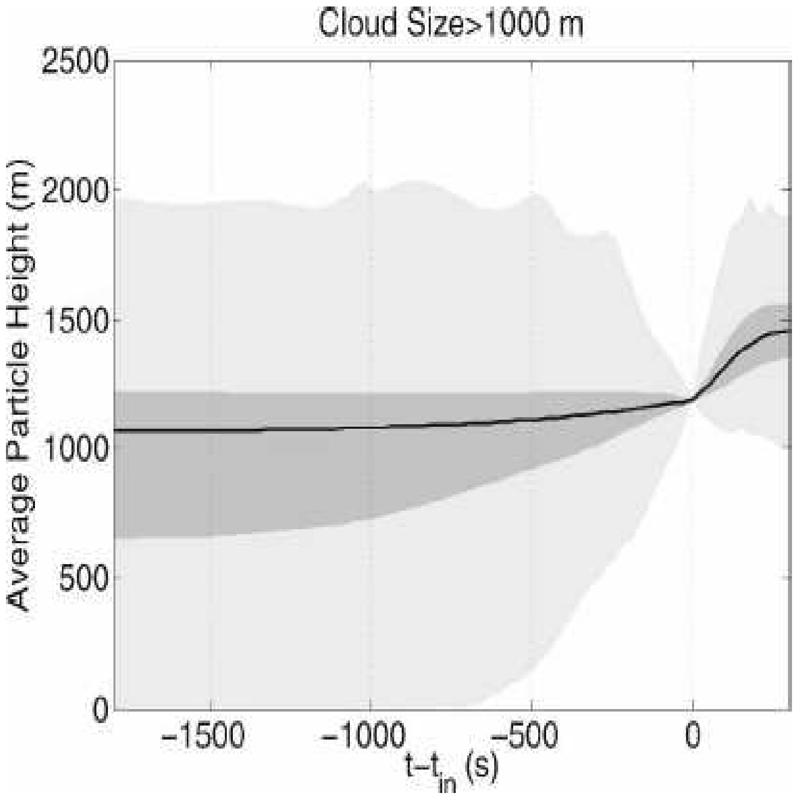
implied source level well above measurement level

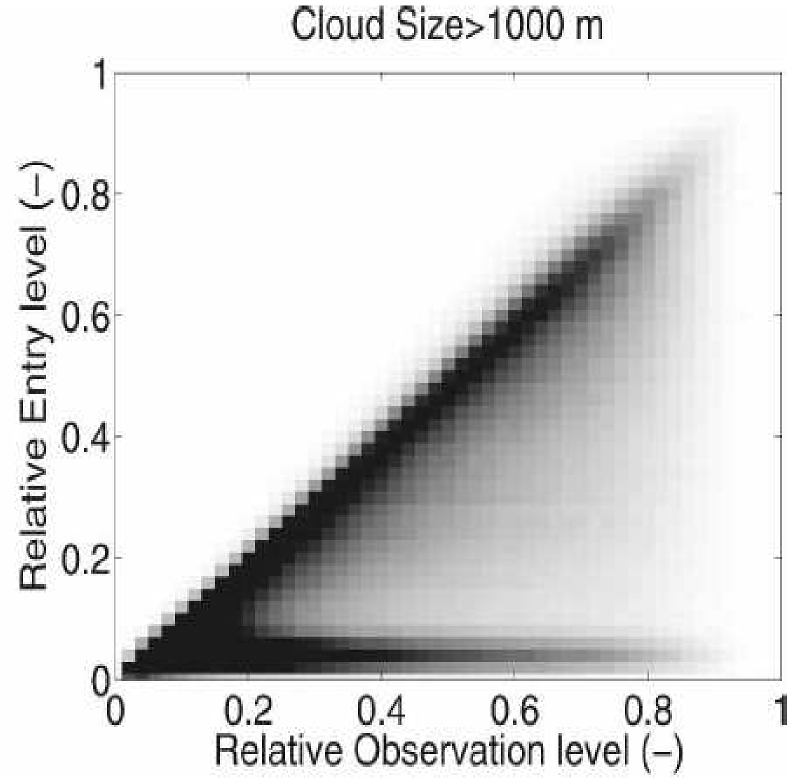
Interpretations of Paluch

- Criticized because data points can line up without implying two-point mixing

  eg, Taylor and Baker 1991; Siebesma 1998
- “On the deceiving aspects of mixing diagrams of deep cumulus convection”
- correlations implied because parcels from below likely to be positively buoyant and those from below negatively bouyant

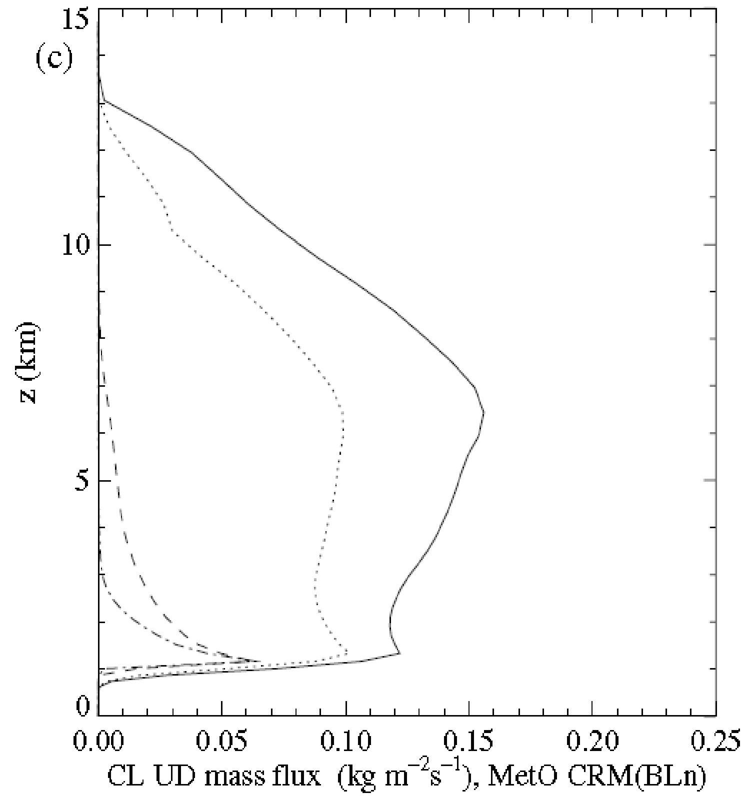
LES Analysis





(Heus et al 2008)

Formulation



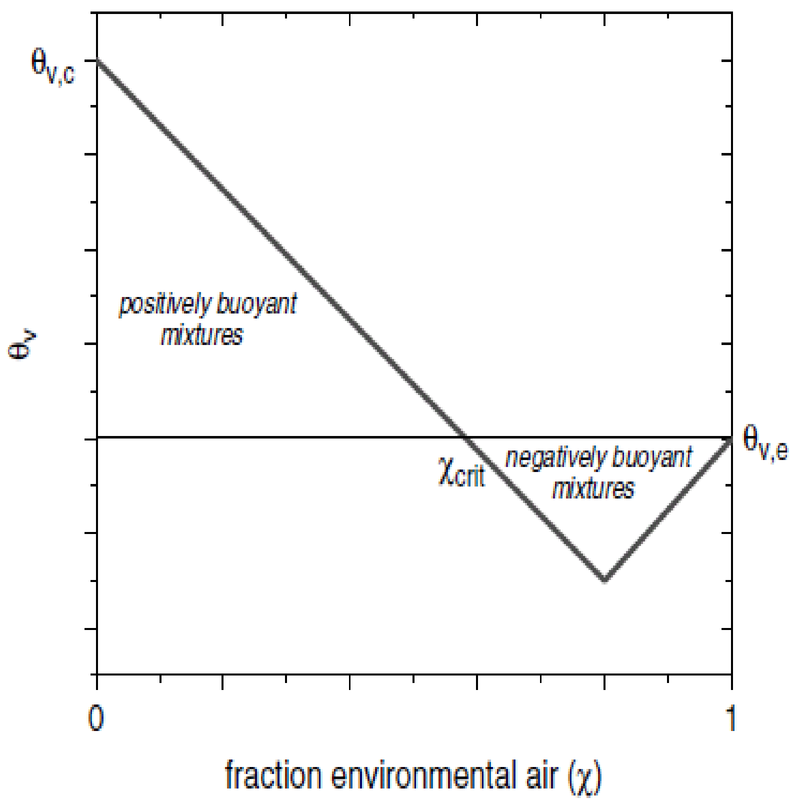
(Derbsyhire et al 2004) sensitivity of CRM to RH

- A lot can be done by formulating E and D as better functions of the environment
- Bechtold et al 2008 revised ECMWF scheme to have entrainment with explicit RH dependence

Stochastic mixing model

- Introduced by Raymond and Blyth (1986) and implemented into Emanuel (1991) scheme
- consider separate parcels from cloud base each of which mixes with air at each level up to cloud top
- mixed parcels spawn further parcels each of which can mix again with air at ecah level from the current one up to cloud top
- can incorporate lateral and cloud-top mechanisms
- how to proportion the air into different parcels?
- Suselj et al (2013) have explicitly stochastic treatment with Poisson process: unit chance of mixing 20% of the mass per distance travelled

Buoyancy Sorting, Kain-Fritsch



- Ensemble of cloud/environment mixtures: retain buoyant mixtures in-plume and detrain negatively buoyant
- evaporative cooling can make mixture $\theta_v <$ environmental θ_v

pdf of mixtures

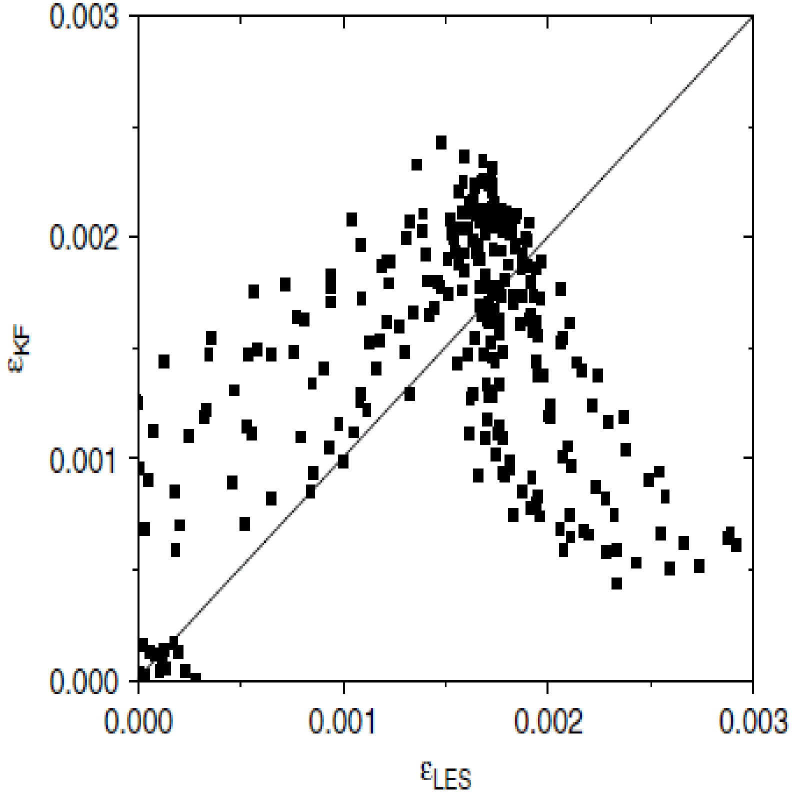
- To complete calculations, also need PDF for occurrence of the various mixtures
- This has to be guessed
- Uniform pdf gives

$$\varepsilon_{KF} = \varepsilon_0 \chi_{crit}^2$$

$$\delta_{KF} = \varepsilon_0 (1 - \chi_{crit})^2$$

where ε_0 is the fraction of the cloud that undergoes some mixing

BOMEX LES estimates



From BOMEX case

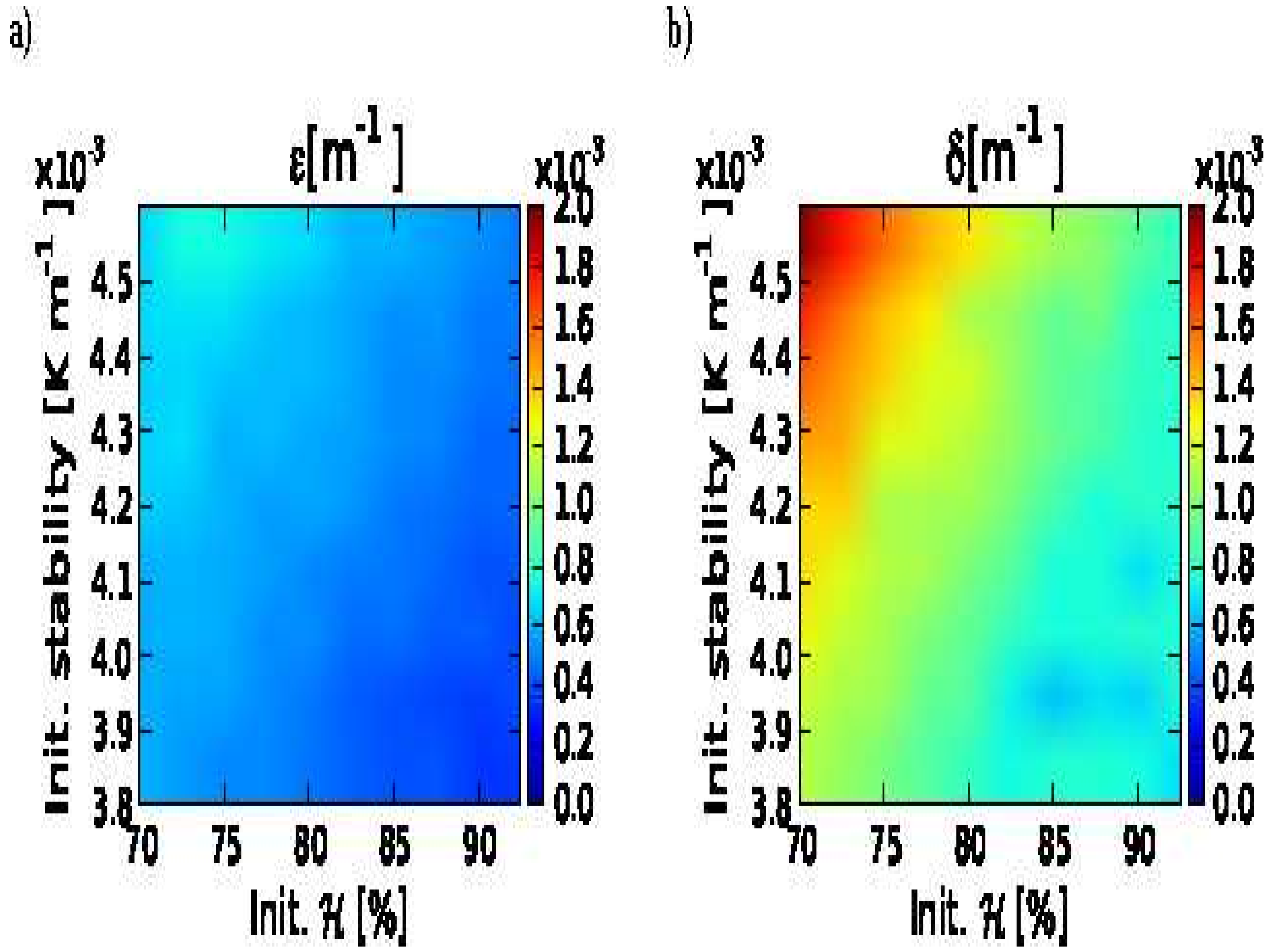
- dry conditions $\longrightarrow$ small $\chi_{crit} \longrightarrow$ weak dilution

$$\varepsilon_{KF} = \varepsilon_0 \chi_{crit}^2$$

- various fixes possible (Kain 2004, Bretherton and McCaa 2004)

The University of Reading

Detrainment variations



Boing et al 2012

Detrainment variations

- Variations of LES estimates dominated by δ not ε
- Variations dominated by cloud-area not by in-cloud w
  (e.g. Derbyshire et al 2011)

Conclusions

- Small clouds are shallower: larger fractional entrainment due to mixing on dimensional grounds
- Some progress on process-level analysis of entrainment and detrainment, but difficult to translate into reliable E and D for use in bulk scheme

  main issue is *how much* of the cloudy material mixes in each way
- Distribution of cloud tops affected by environment
- This controls the organized detrainment contribution
- which seems to be an important control on the overall bulk profile

Closure

Objective

We need to calculate the total mass flux profile,

$$M = \sum_i M_i = \eta(z) M_B(z_B)$$

- $\eta(z)$ comes entrainment/detrainment formulation
- $M_B = M(z_B)$ remains, the overall amplitude of convection

Practical Issue

- A practical convection scheme needs to keep the parent model stable

  Settings may err on the defensive side to remove potential instability

- not all diagnostic relationships for M_B are appropriate

$$M_B = k \frac{C_p \overline{w'T'}_0 + L\overline{w'q'}_0}{\mathrm{CAPE}}$$

  Shutts and Gray 1999

- scaling works well for a set of equilibrium simulations, but not as closure to determine M_B

Convective Quasi-Equilibrium

- Generation rate of convective kinetic energy defined per unit area

$$\int_{z_B}^{z_T} \sigma\rho w_c b dz \equiv M_B A$$

- where the “cloud work function” is

$$A = \int_{z_B}^{z_T} \eta b dz.$$

- For each plume type

$$A(\lambda) = \int_{z_B}^{z_T(\lambda)} \eta(\lambda, z) b(\lambda, z) dz.$$

Convective Quasi-Equilibrium

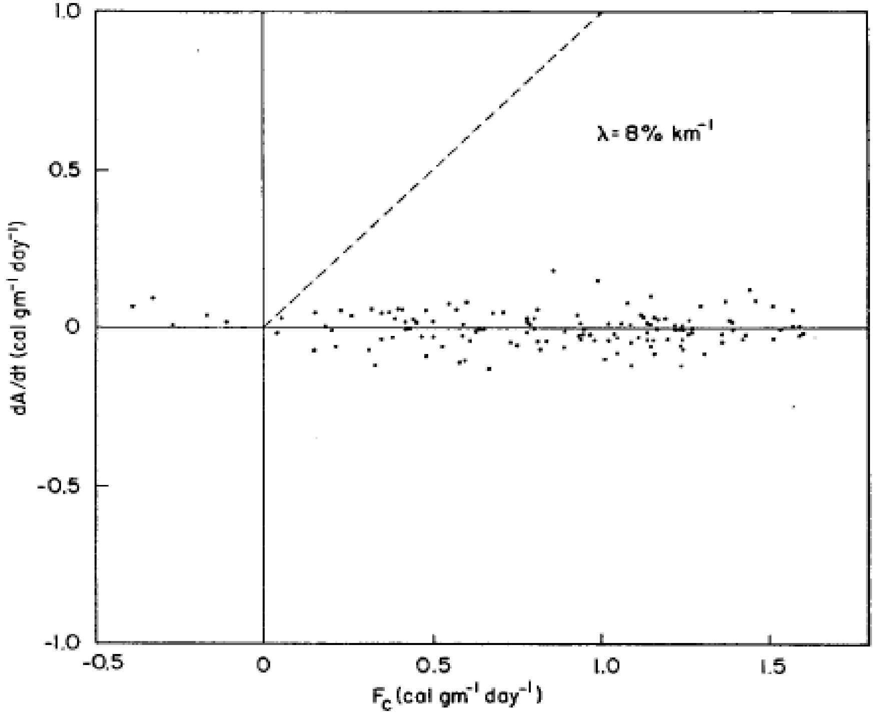
- Taking a derivative of the definition

$$\frac{\partial}{\partial t}A_\lambda = F_{L,\lambda} - D_{c,\lambda}$$

  where

- $F_{L,\lambda}$ is “large-scale” generation: terms independent of M_B
- $D_{c,\lambda}$ is consumption by convective processes: terms dependent on M_B, proportional for entraining plumes with simplified microphysics in AS74
- “scale” not immediately relevant to this derivation which follows by definition
- all of the cloud types consume the CWF for all other types

Convective Quasi-Equilibrium



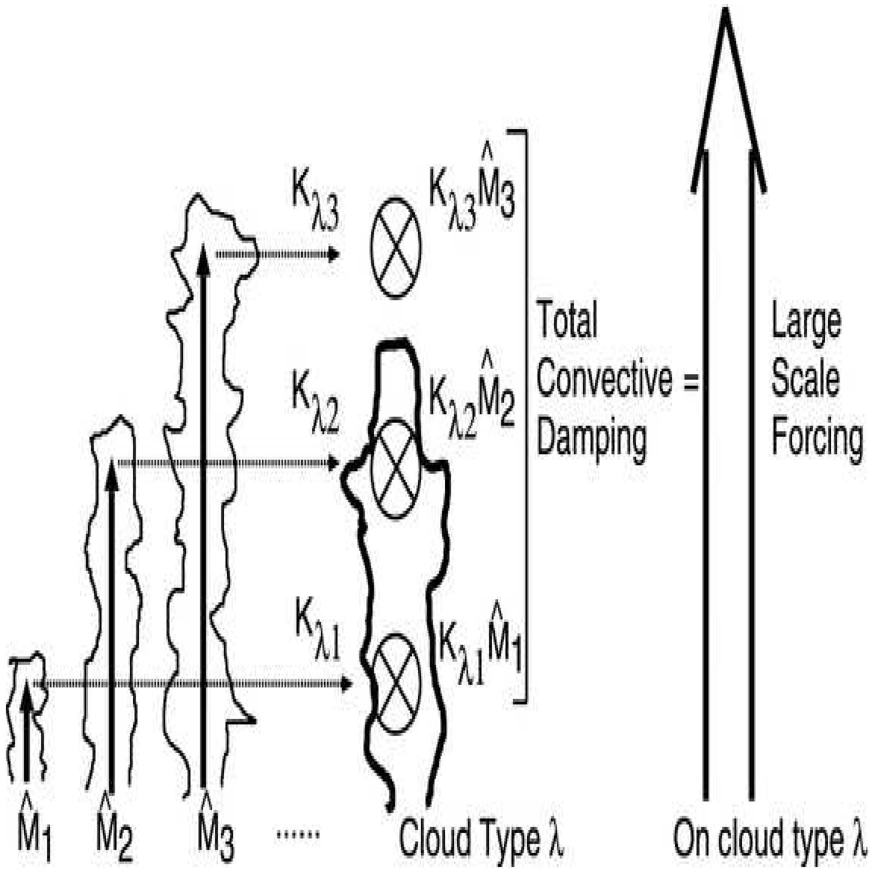
A stationary solution to the CWF tendency equation

$$F_{L,\lambda} - D_{c,\lambda} = 0$$

$$\sum_{\lambda'} \mathcal{K}_{\lambda\lambda'} M_{B,\lambda'} = F_{L,\lambda}$$

Assumes $\tau_{\rm LS} \gg \tau_{\rm adj}$

Using CQE



$$\sum_{\lambda'} \mathcal{K}_{\lambda\lambda'} M_{B,\lambda'} = F_{L,\lambda}$$

- $F_{L,\lambda}$ is known from parent model
- $\mathcal{K}_{\lambda\lambda'}$ is known from the plume model
- invert matrix $\mathcal{K}$ to get $M_{B,\lambda}$

Issues with CQE calculation

1. The resulting $M_{B,\lambda}$ is not guaranteed positive
   various fixes possible, eg Lord 1982; Moorthi and Suarez 1992
2. the equilibrium state is not necessarily stable
3. $\eta(z,\lambda)$ and $b(z,\lambda)$ depend on $T(z)$ and $q(z)$. If the $A(\lambda)$ form a near-complete basis set for T and q, then stationarity of all A would imply highly- (over-?) constrained evolution of T and q

Some CWF variants

$$A(\lambda) = \int_{z_B}^{z_T(\lambda)} \eta(\lambda, z) b(\lambda, z) dz$$

1. CAPE$= A(\lambda = 0)$, ascent without entrainment
2. CIN: negative part of integated non-entraining parcel buoyancy
3. Diluted CAPE: ascent with entrainment, but differs from CWF by taking $\eta = 1$ in integrand
4. PEC (potential energy convertibility): bulk A estimate by choosing a different normalization
5. Other quantities investigated based on varying the limits of the integral

   (e.g. “parcel-environment” CAPE of Zhang et al 2002, 2003)

CAPE closure

- CAPE is a special case of $A(\lambda)$ for zero entrainment
- So its quasi-equilibrium is based on $\tau_{\mathrm{LS}} \gg \tau_{\mathrm{adj}}$
- We could close a spectral scheme using CAPE plus some other way of setting the spectral distribution
- We could close a bulk scheme directly using CAPE

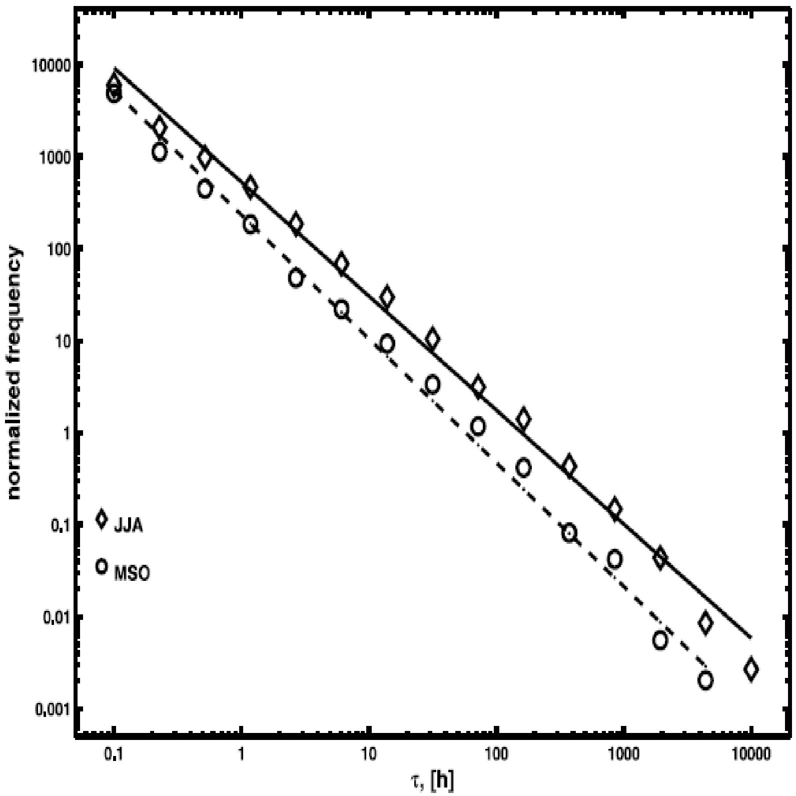
CQE Validity

- Zimmer et al (2010) timescale for CAPE consumption rate

$$\tau \sim \mathrm{CAPE}/P$$

assuming precipitation rate $P \sim (d\mathrm{CAPE}/dt)_{\mathrm{conv}}$

- P is average within 50 km radius and 3 hr window
- 2/3 of events have less than 12 hours



Operational CAPE closure

- In many operational models assumed that convection consumes CAPE at a rate that is determined by a characteristic closure time–scale τ_c.

$$M_B \propto \left. \frac{dCAPE}{dt} \right|_{\mathrm{conv}} = -\frac{CAPE}{\tau_c}$$

(Fritsch and Chappell 1980)

- Conceptually, maintains idea of timescale separation, but recognizes finite convective-consumption timescale
- Many variations on this basic theme:
- As well as variations of the CAPE-like quantity, some experiments with a functional form for τ_c

Moisture-based closure

- large–scale supply of moisture balanced against consumption by convective processes
- some methods consider only large–scale convergence, but others add surface fluxes
- remains a popular approach since original proposal by Kuo 1974
- especially for applications to models of tropical deep convection
- Emanuel 1994, causality problem assuming convection is driven by moisture rather than by buoyancy
- tendency for grid–point storms

PBL-based closures

- Mapes 1997 deep convection may be controlled by:
  - equilibrium response to increases in instability
  - the ability to overcome CIN (activation control)
- On large-scales, CIN will always be overcome somewhere and equilibrium applies
- On smaller scales, PBL dynamics producing eddies that overcome CIN may be important
- Mapes 2000 proposed $M_B \sim \sqrt{\mathrm{TKE}}\,\exp(-k\mathrm{CIN}/\mathrm{TKE})$
- To be discussed!

Which is right?

- Buoyancy-based, moisture-convergence-based and PBL-based methods all have some intuitive appeal
- Analyses are bedevilled by “chicken-and-egg” questions
- Convection “consumes” moisture and CAPE on the average, but not always, and the exceptions matter
- e.g., shallow convection
- Various analyses attempt to correlate rainfall (note not M_B!) with various factors
  - results, while interesting, are typically not conclusive
  - and correlations typically modest (or even anti!)
  - and different for different regions

(Sherwood and Warlich 1999, Donner and Phillips 2003, Zhang et al 2002, 2003, 2009, 2010)

Conclusions

- Cloud work function is a measure of efficiency of energy generation rate
- CAPE is a special case, as are various other measures
- Quasi-equilibrium if build-up of instability by large-scale is slow and release at small scales is fast
- Similar QE ideas can be formulated for the variants, and for moisture
- QE is a often a good basis for a closure calculation, but is not always valid, and may not be a good idea to apply it very strictly