Convection Closure and Energy Cycle

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Outline

- Equilibrium convection closure
- Prognostic equation for cloud work function
- Prognostic equation for convective kinetic energy
- Closing the energy cycle
- Examples of the energy cycle

Equilibrium convection closure

Closure in the mass flux approach

- Convection characterised by plumes, the equations for these being formulated in terms of mass flux, $M = \rho\sigma_c w_c$
- We can decompose the mass flux as

$$M(z) = \eta(z) M_B$$

- $M_B = M(z_B)$ is the cloud base mass flux
- $\eta(z)$ comes from the “cloud model” eg, entraining plume
- M_B needs a “closure” to give us the overall amplitude of convection

Practical Issue

- A practical convection scheme needs to keep the parent model stable

  Settings may err on the defensive side to remove potential instability

- not all diagnostic relationships for M_B are appropriate

$$M_B = k\frac{C_p\overline{w'T'}_0 + L\overline{w'q'}_0}{\mathrm{CAPE}}$$

  Shutts and Gray 1999

- scaling works well for a set of equilibrium simulations, but not as closure to determine M_B
- Models may produce on/off behaviour, `www.met.rdg.ac.uk/~sws00rsp/anim/timestep.html`

A general formulation

- Consider some function f of the large-scale variables φ and the convective-scale variables φ_c and η
- Integrate this over some range of heights,

$$I = \int f(\varphi, \varphi_c, \eta) dz$$

- We can make a closure condition through stationarity of this quantity, $\partial I / \partial t = 0$
- The only requirement is that φ within the integral range is affected by the amplitude of convection

General structure

- Take a time derivative of the definition, and substitute for $\partial\varphi/\partial t, \partial\varphi_c/\partial t$ and $\partial\eta/\partial t$ using equations developed from the mass flux framework
- After some algebra, the result has the schematic form

$$\frac{\partial I}{\partial t} = F - D$$

- F is large-scale generation or “forcing”: terms independent of M_B
- D is consumption by convective processes: terms dependent on M_B, proportional for entraining plumes with simple microphysics
- NB: scale not immediately relevant to the derivation

Example: Moisture Closure

- This can be obtained by choosing $f = \rho q$
- We find:

$$F = \text{moisture convergence} = -\int_0^{z_T} \nabla.\rho\mathbf{u}q dz + E$$

$$D = M_B \int_{z_b}^{z_T} \eta \left[\delta_c (q_c - q) + \frac{\partial q}{\partial z}\right] dz$$

Moisture-based closure

- Large-scale supply of moisture balanced against consumption by convective processes
- Some methods consider only large-scale convergence, but others add surface fluxes
- Remains a popular approach since original proposal by Kuo 1974
- Especially for applications to models of tropical deep convection
- Shallow convection under large-scale descent is obvious counterexample
- Tendency for grid–point storms

Other examples

- CAPE closure with $f = b$, the buoyancy for non-entraining parcel ascent
- Parcel-environment closure by considering only the contributions to b from changes to free tropospheric variables
- PCAPE closure combines this with the choice $f = \rho b$, as recently developed by Bechtold et al (2014) for the new ECMWF closure

Cloud work function

- Generation rate of convective kinetic energy per unit area

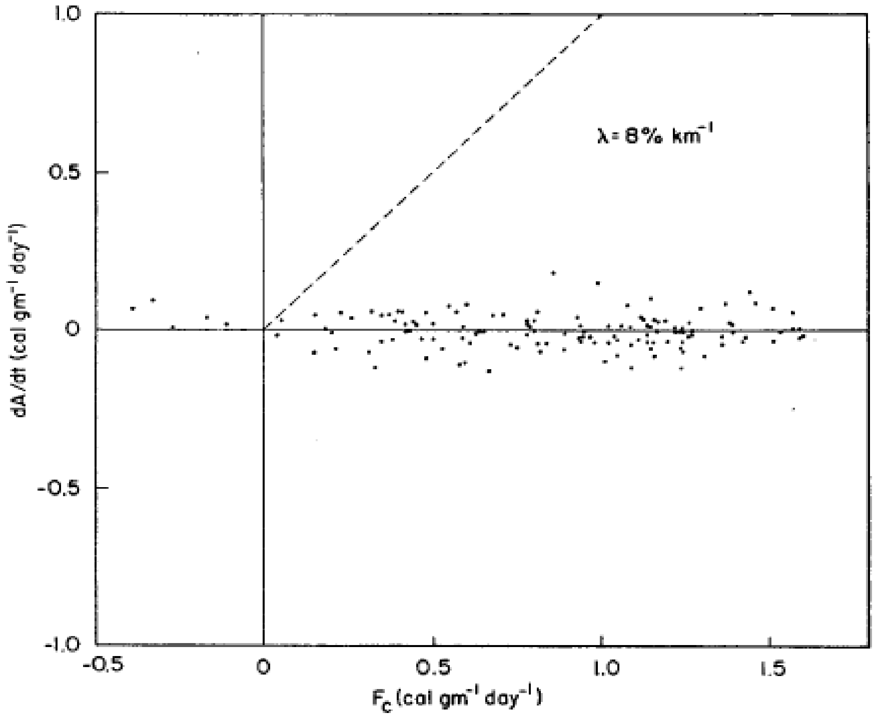
$$\int_{z_B}^{z_T} \rho\sigma_c w_c b dz \equiv M_b A$$

- where the "cloud work function" is

$$A = \int_{z_B}^{z_T} \eta b dz$$

- This matches our general framework with $f = \eta b$
- Note that $b(z)$ and z_T will depend on entrainment assumptions as well as $\eta(z)$
- CAPE$= A(\lambda = 0)$, ascent without entrainment

Convective Quasi-Equilibrium for A



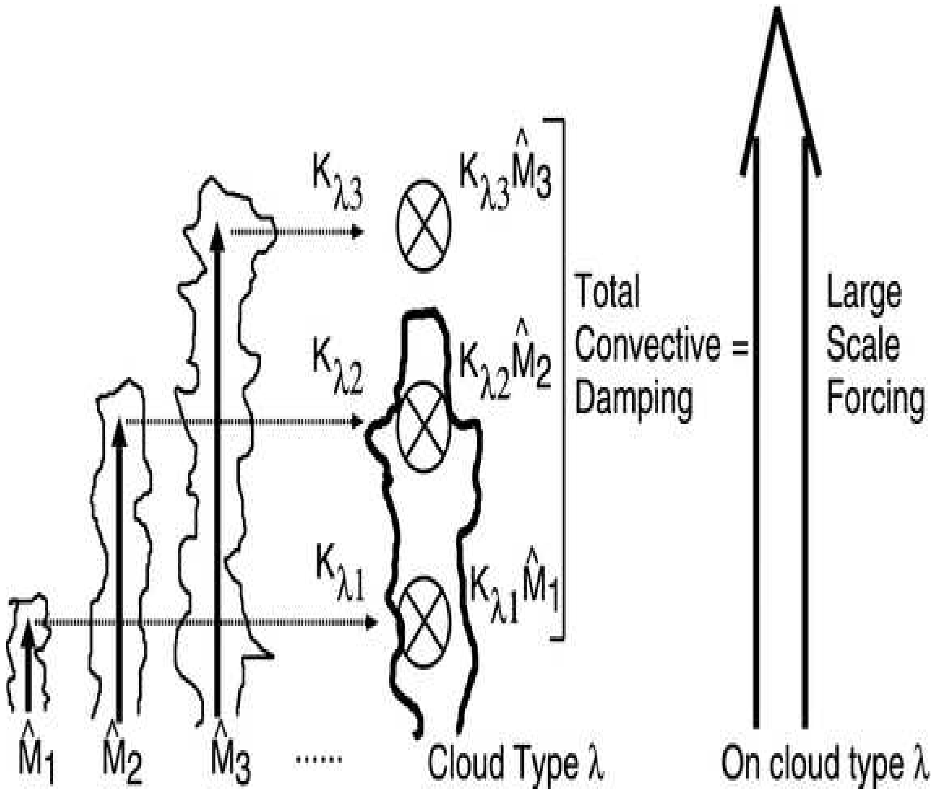
A stationary solution to the CWF tendency equation

$$\frac{\partial A_i}{\partial t} = F_i - D_i \approx 0$$

where

$$D_i = \sum_j \mathcal{K}_{ij} M_{B,j}$$

Using CQE for closure



$$\sum_j \mathcal{K}_{ij} M_{B,j} = F_i$$

- F_i is known from the GCM / NWP
- $\mathcal{K}_{ij}$ is known from the plume model
- Invert matrix $\mathcal{K}$ to get $M_{B,j}$

Issues with CQE calculation

1. The resulting $M_{b,i}$ is not guaranteed positive
   various fixes possible, eg Lord 1982; Moorthi and Suarez 1992
2. The equilibrium state is not necessarily stable
3. $\eta_i(z)$ and $b_i(z)$ depend on $T(z)$ and $q(z)$. If the A_i form a near-complete basis set for T and q, then stationarity of all A_i would imply highly- (over-?) constrained evolution of T and q

Relaxation approach for bulk schemes

- In many operational models assumed that convection consumes CAPE at a rate that is determined by a characteristic closure timescale τ_c.

$$D = KM_B$$

$$\left.\frac{dCAPE}{dt}\right|_{\text{conv}} = -\frac{CAPE}{\tau_c}$$

- If forcing almost constant, this relaxes towards an equilibrium
- Many variations of the CAPE-like quantity and various experiments with functional forms for τ_c

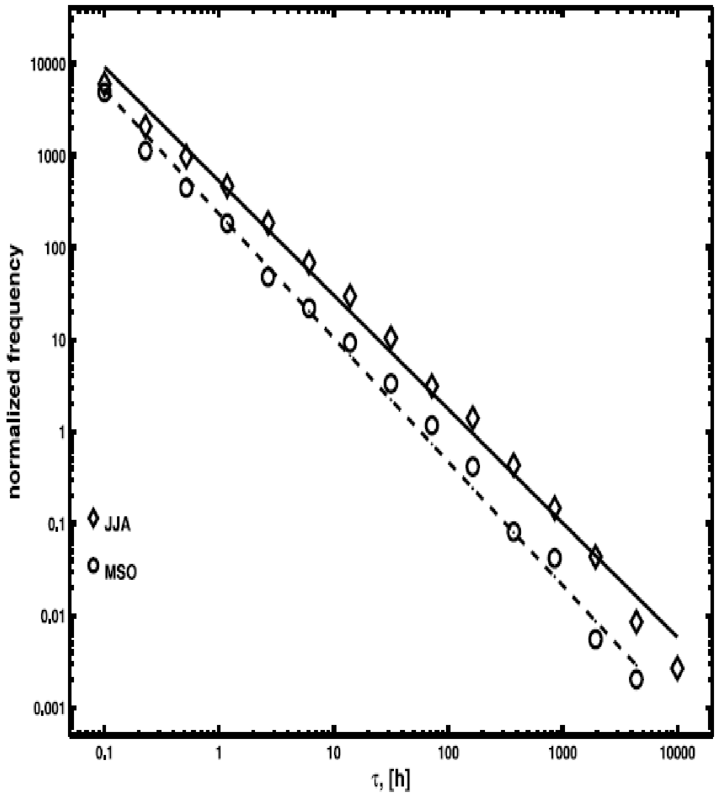
CQE Validity

- Zimmer et al (2010) timescale for CAPE consumption rate

$$\tau \sim \mathrm{CAPE}/P$$

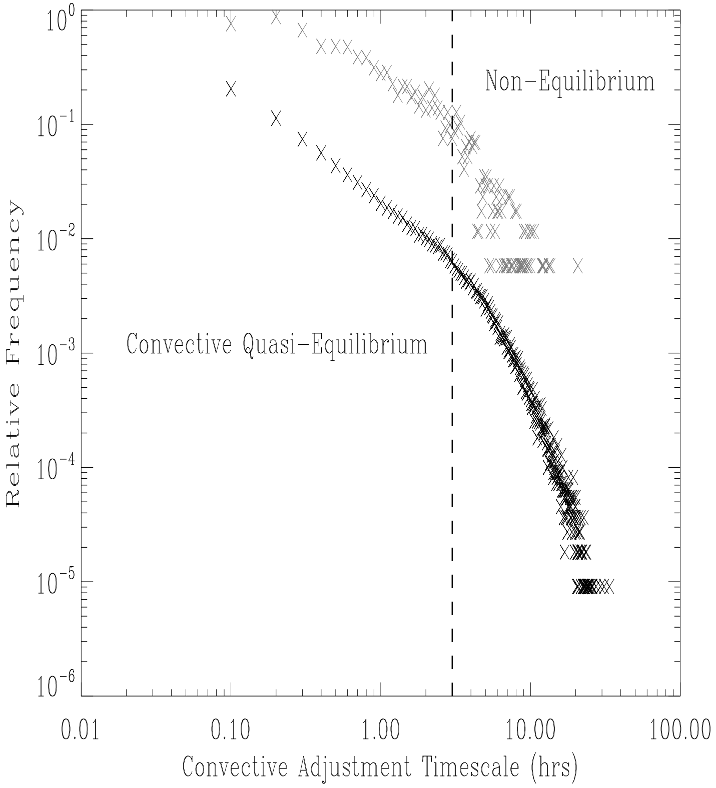
  assuming precipitation rate $P \sim (d\mathrm{CAPE}/dt)_{\mathrm{conv}}$

- P is average within 50 km radius and 3 hr window
- Uses combination of sonde and radar data in Germany



CQE Validity

- Results from UKV model output...
- Similar slope for $\tau < 3$ hr but falls off more rapidly at higher τ
- Conditions over the UK and/or in explicit convection permitting models are more equilibrium-like



What's wrong with convection in the UM!?

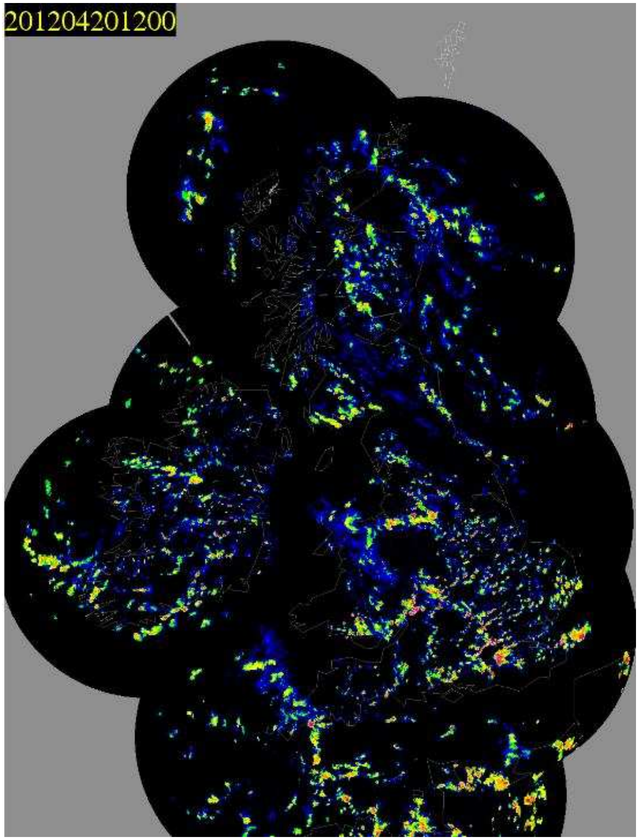
- Explicit simulations with resolutions in the 1-10 km range tend to produce unrealistically-intense cells; either grid-scale (not really resolved) or artificially large.

- Need to represent unresolved fluxes from partially / marginally-resolved convection.

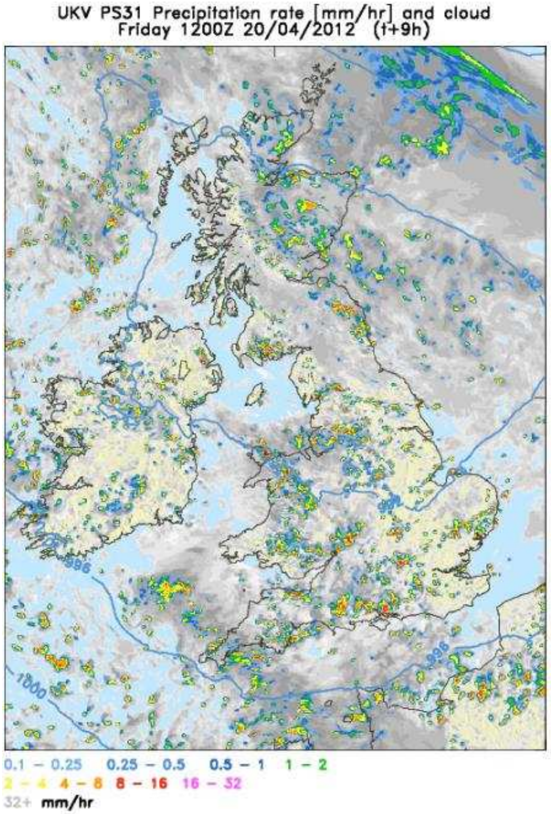
- But introducing current convection parameterisations makes things worse!

Example from UK convective-scale forecast

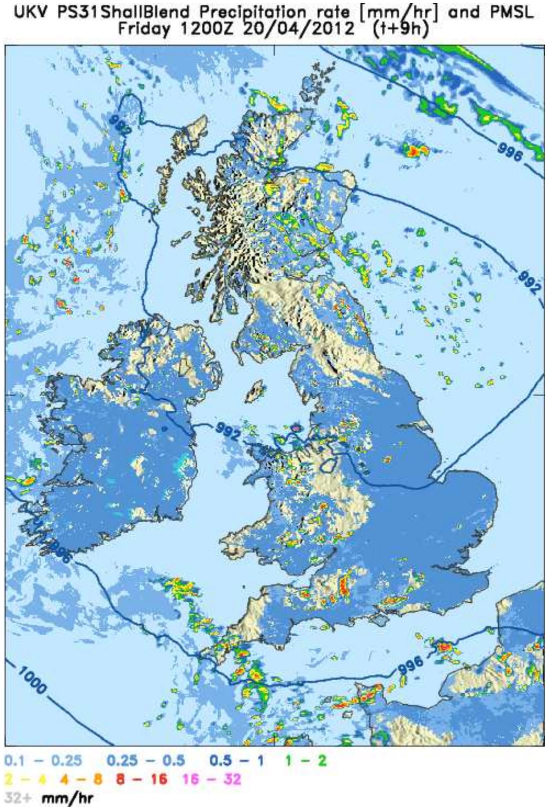
Radar



1.5 km explicit



1.5 km parameterised



Prognostic equation for cloud work function

Energy cycle

- From taking a derivative of the definition, recall that

$$\frac{\partial A_i}{\partial t} = F_i - \sum_j \mathcal{K}_{ij} M_{b,j}$$

- We noted that A is an important quantity for the generation of convective kinetic energy
- We now look how that works...

Prognostic equation for convective kinetic energy

Convective kinetic energy

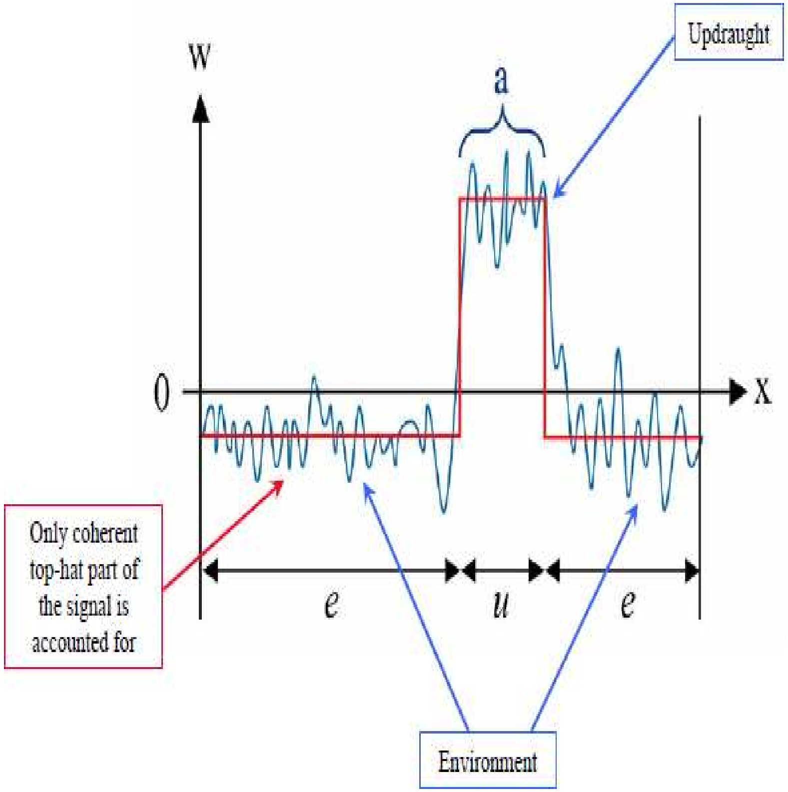
- This is $K_v = (1/2)\int_{z_b}^{z_T} \rho\sigma_c w_c^2 dz$
- Or we could consider $K_3 = (1/2)\int_{z_b}^{z_T} \rho\sigma_c(u_c^2 + v_c^2 + w_c^2)$
- In either case we will find that the convective kinetic energy equation is

$$\frac{dK}{dt} = AM_b - L$$

- where L is a loss of convective kinetic energy ("dissipation") with a formula that depends on the definition taken for K

Equation for $\partial K/\partial t$

- Derivation starts from the decomposition of space into convective and environmental parts
- Within each, apply the segmentally-constant approximation
- Ths, the convective velocity w_c is that obtained from a horizontal average over the areas identified as convective



Equation for $\partial K/\partial t$

This leads to

$$\frac{\partial \sigma_c w_c}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial z}\rho\sigma_c w_c^2 + \frac{1}{S}\oint_{\partial S} w_b(\mathbf{u}_b^* - \dot{\mathbf{r}}_b)\cdot d\mathbf{r} = \boldsymbol{\sigma}\left[-\frac{1}{\rho}\frac{\partial p'}{\partial z} + b\right]$$

Multiplying by w_c and using mass continuity this produces

$$\frac{\partial \sigma_c k_v}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial z}\rho\sigma_c w_c k_v + (E+D)k_v = -\frac{M}{\rho}\frac{\partial p'}{\partial z} + Mb$$

where $k_v = (1/2)\rho w_c^2$

Equation for $\partial K/\partial t$

The final step is to integrate over height

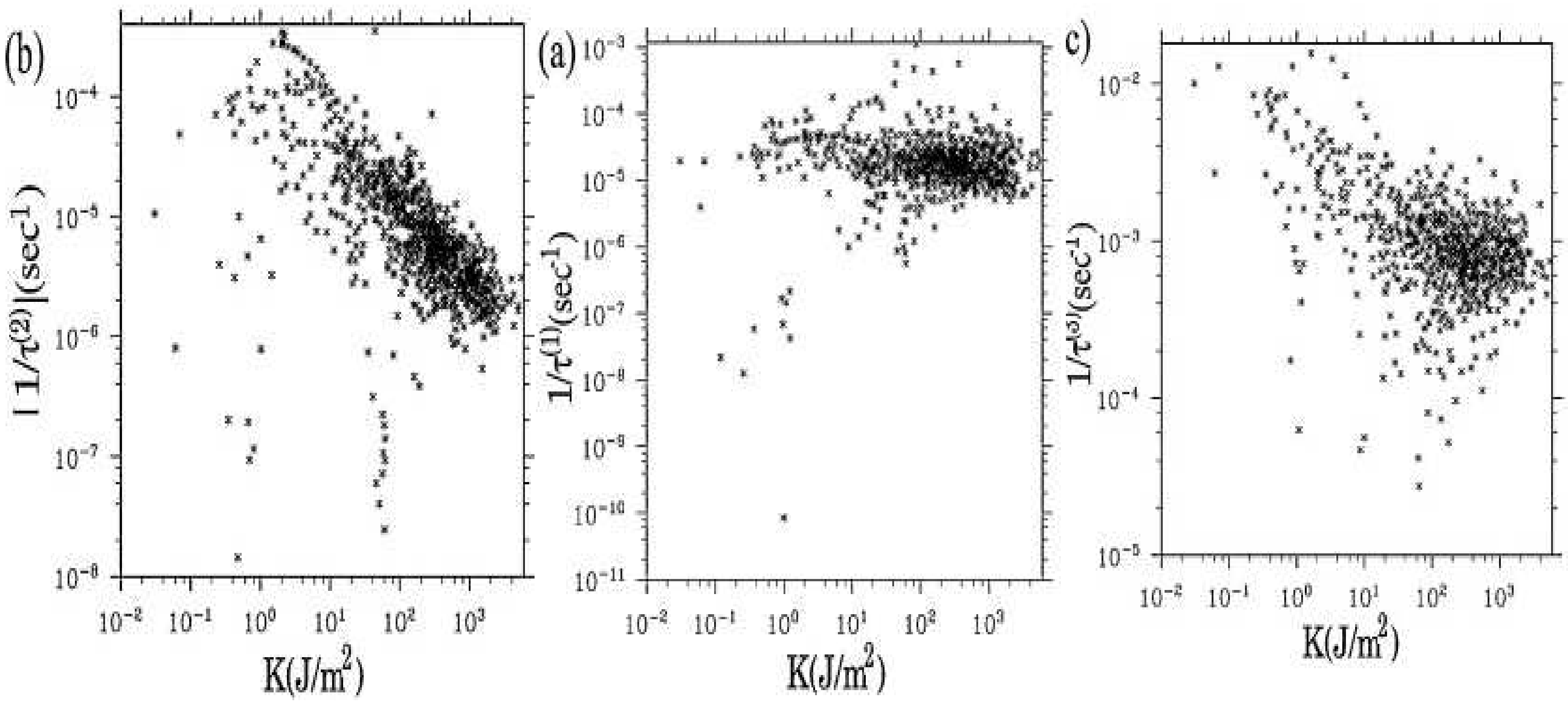
$$\frac{\partial K_v}{\partial t} = AM_b - L$$

with

$$L = \left[\frac{M}{\rho}k_v\right]_{z_b}^{z_T} + \int_{z_b}^{z_T}(E+D)\frac{K_v}{\rho}dz + \int_{z_b}^{z_t}\frac{M}{\rho}\frac{\partial p'}{\partial z}dz$$

So that the loss CKE can be produced by: fluxes through base and top of cloud, exchange across cloud edges and a pressure term

Contributions to L



Top/bottom term (left), entrainment/detrainment term (centre) and pressure term (right)

Equation for $\partial K_3/\partial t$

- A similar analysis shows that

$$\frac{\partial K_3}{\partial t} = AM_b - L_3$$

- with additional contributions arising in the loss term L_3, most notably in the pressure-related terms

Convective kinetic energy equation

- In conclusion we find that

$$\frac{\partial K_3}{\partial t} = AM_b - L_3$$

- and it seems to be reasonable to write the loss term in the form

$$L = \frac{K}{\tau_D}$$

Closing the energy cycle

Non-equilibrium, possible motivations

- For relatively rapid forcings, we may wish to consider a prognostic equation for cloud-base mass flux
- Even for steady forcing, this may be more convenient than a matrix inversion if there are various types
- Even for steady forcing, it is not obvious
  - that a stable equilibrium must be reached
  - which equilibrium might be reached

Non-equilibrium by Pan and Randall

- They start from

$$\frac{dA_i}{dt} = F_i - \sum_j K_{ij} M_j$$

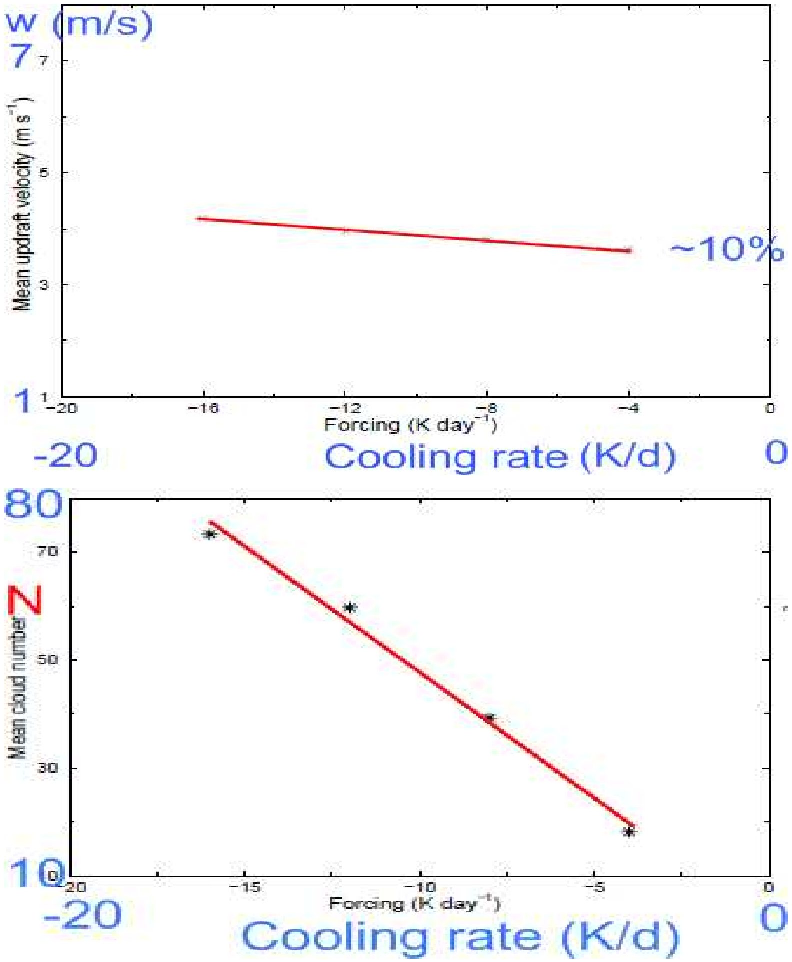
$$\frac{dK_i}{dt} = A_i M_i - \frac{K_i}{\tau_D}$$

and they introduced the assumption

$$K_i = \alpha M_i^2$$

- Plausible as $K \sim \sigma_c w_c^2$ and $M = \rho \sigma_c w_c$; assumes variations in w_c dominate variations in K and M
- A damped linear oscillator that approaches equilibrium after a few τ_D

CRM with changes to imposed forcing



Increased forcing linearly increases the mass flux, $\rho\sigma w$

- achieved by increasing cloud number $\langle N \rangle$
- not the in-cloud velocities
- nor the sizes of clouds

(Cohen 2001)

Non-equilibrium of Plant and Yano

- They start from

$$\frac{dA_i}{dt} = F_i - \sum_j K_{ij} M_j$$

$$\frac{dK_i}{dt} = A_i M_i - \frac{K}{\tau_D}$$

but now use the assumption

$$K_i = \beta M_i$$

- Plausible as $K \sim \sigma_c w_c^2$ and $M = \rho \sigma_c w_c$; assumes variations in σ dominate variations in K and M

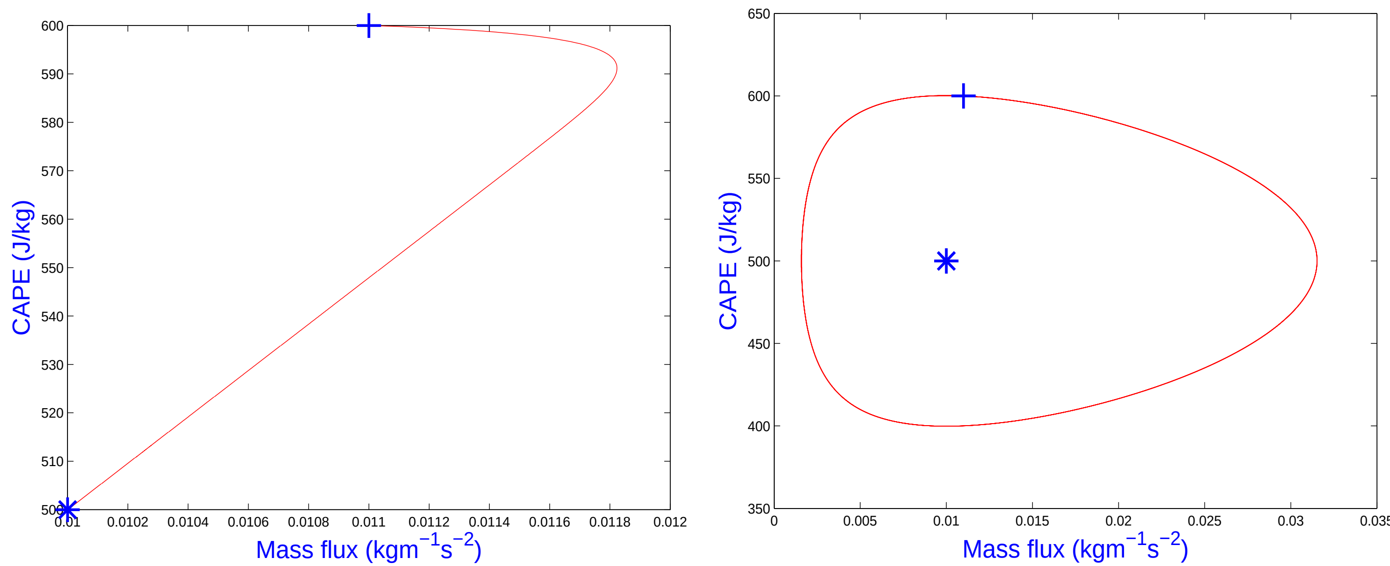
Non-equilibrium of Plant and Yano

- Consistent with various scalings and CRM data for changes in mass flux with forcing strength

  Emanuel and Bister 1996; Robe and Emanuel 1996; Grant and Brown 1999; Shutts and Gray 1999; Cohen 2001; Parodi and Emanuel 2009; Davies 2009
- e.g., The CQE solution has
  - CAPE independent of F if $K = \beta M$
  - CAPE$\propto F$ if $K = \alpha M^2$
- CRMs are much closer to the first of these

Examples of the energy cycle

Illustrative results for $K \sim M^p$



Pan & Randall, $p = 2$ and Yano & Plant, $p = 1$ systems

$$x - \ln(1 + x) + \frac{y^2}{2} = C$$

for $p = 1$ with x the rescaled mass flux and y the rescaled CWF

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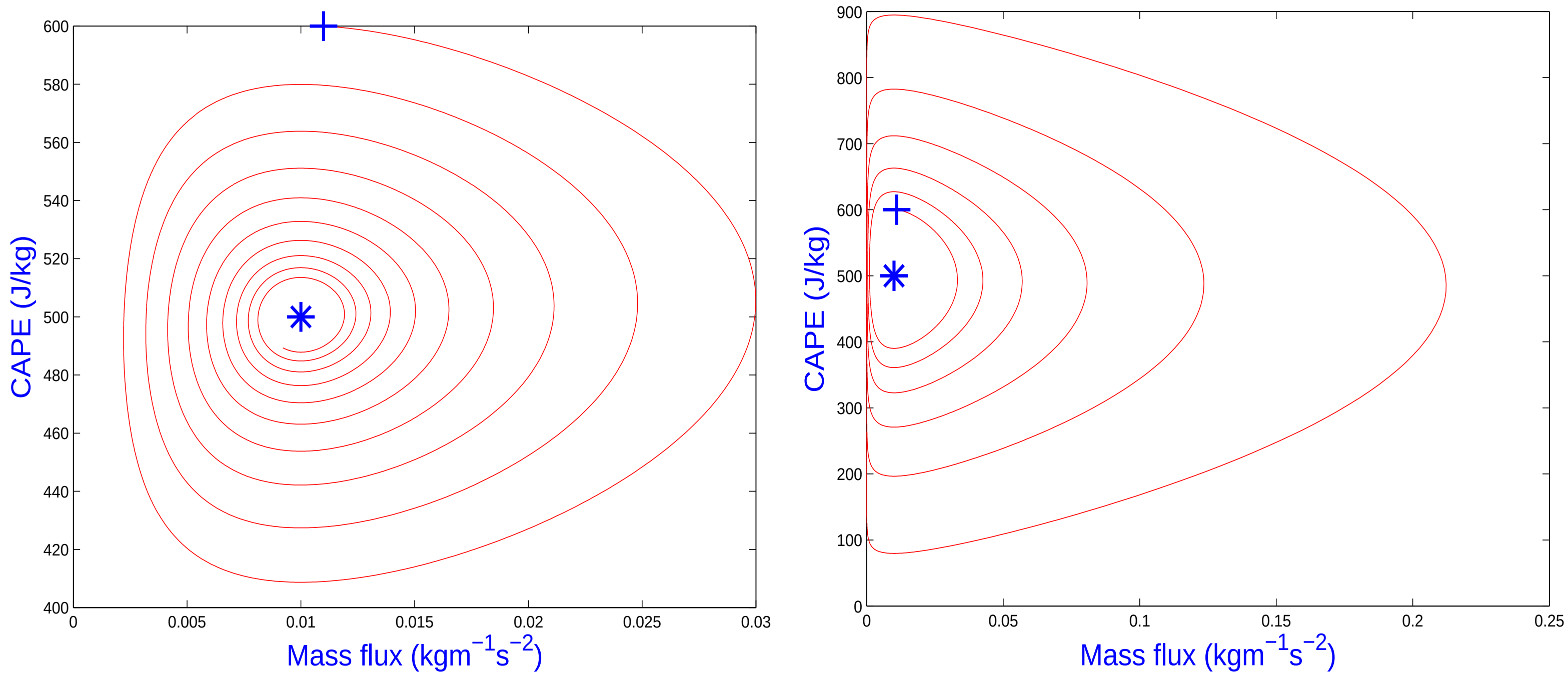
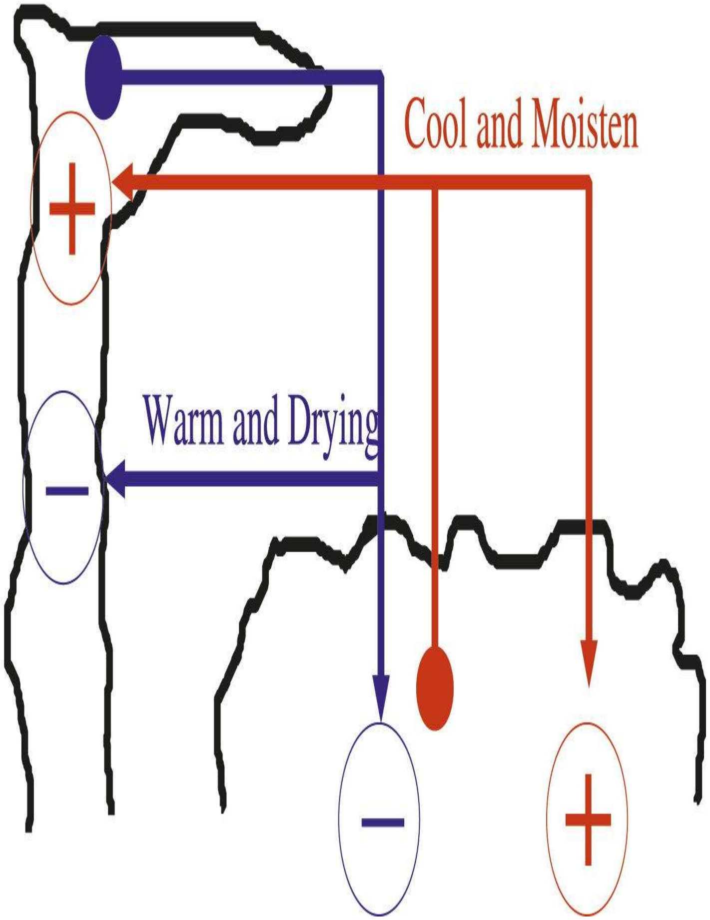
Illustrative results for $K \sim M^p$



$p = 1.01$ (left) and $p = 0.99$ (right)

- The CRM data supports $p \approx 1$ but > 1
- Equilibrium is reached but more slowly as $p \longrightarrow 1$ from above

Interactions of two types



- Deep convection consumes instability and damps all convection types
- In some situations, shallow convection can pre-condition atmosphere for deep convection, giving +ve feedbacks
- Can describe this with energy cycle system

Possible solutions with two types

With no external forcing...

- Deep convection dominates and all convection ultimately dies out
- Shallow convection dominates and system explodes
- Shallow and deep are well-blanced, giving self-sustaining solution?

Analytic conditions for solutions

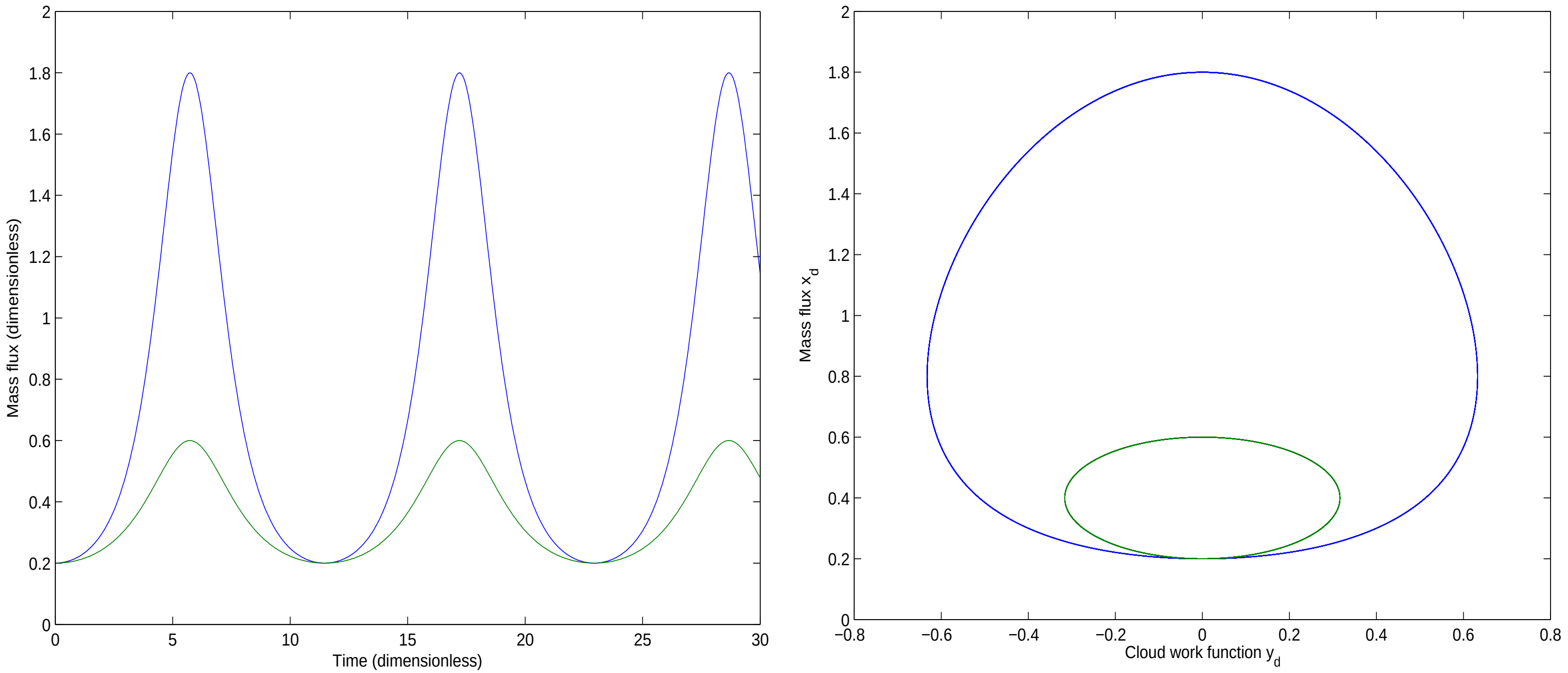
With $p = 2$ we get periodic solution if:

- Determinant of K matrix vanishes
- Generation of CWFs by shallow weaker than destruction by deep
- $\tau_d > \tau_s$, deep is damped more strongly than shallow

With $p = 1$ a nonlinear self-sustaining solution can also occur

- Conditions are more complicated, but can still be derived analytically

Illustrative solution of two types



$p = 1$ with no external forcing
Blue for deep and green for shallow

In summary

- CQE in its original form is based on energy cycle equations for ensemble of plumes
- Most operational schemes do not apply CQE strictly but relax towards an equilibrium
- With an extra assumption to link K, M_B and A, energy cycle becomes a prognostic system
- Simple assumptions used so far are not perfect but can produce some interesting behaviours
- Could use analytic conditions derived from non-equilibrium system even in an equilibrium context
  - eg, to decide if equilibrium has no convection, shallow only, deep only or both

Smaller convective ensembles
(If time and interest)

Smaller convective ensembles

- A crucial point to remember is that all of the above applies to convective ensembles, and not to the evolution of individual clouds
- Can the description be generalized to situations to situations where there are likely to be multiple clouds present but not necessarily very many?
- Can be achieved with a system-size expansion of a simple probabilistic model

Ingredients

- $P(N, A, t)$ is pdf for N clouds and cloud work function A at time t
- Consider domain with size Ω elements, each of which may be either empty or occupied by a single cloud
- P evolves through master equation

$$\frac{\partial P(N,A,t)}{\partial t} = \int dA' \sum T(N,A|N',A')P(N',A',t)$$

$$-T(N',A'|N,A)P(N,A,t)$$

- where $T(f|i)$ is probability per unit time of a transition from i to f

Transition Rules

- At each time, look at one site with probability $1-\mu$ or two with probability μ
- Suppose we look at one. With probability $1-(N/\Omega)$ it is empty
- Suppose it is empty:
  - With probability a we allow cloud formation here, $N \longrightarrow N+1$
  - Otherwise it remains empty and atmosphere continues to be destabilized, $A \longrightarrow A+s$
- e.g. for spontaneous birth we have

$$T(N+1,A|N,A') = a(1-\mu)\left(1-\frac{N}{\Omega}\right)\delta(A-A')$$

Possible Processes

$E \xrightarrow{a} O$	$A \rightarrow A$	spontaneous birth (primary initiation)
$E \xrightarrow{1-a} E$	$A \rightarrow A+s$	environmental destabilization
$O \xrightarrow{d} E$	$A \rightarrow A$	death
$O \xrightarrow{1-d} O$	$A \rightarrow A-r$	environmental stabilization
$EO \xrightarrow{b} OO$	$A \rightarrow A$	induced birth (secondary initiation)
$EO \xrightarrow{1-b} EO$	$A \rightarrow A+s-r$	environmental modification
$OO \xrightarrow{c} EO$	$A \rightarrow A$	competitive exclusion
$OO \xrightarrow{1-c} OO$	$A \rightarrow A+2s$	strong stabilization
$EE \xrightarrow{e} EO$	$A \rightarrow A$	birth
$EE \xrightarrow{1-e} EE$	$A \rightarrow A-2r$	strong destabilization

System size expansion

- Expand the master equation for P in powers of $1/\sqrt{\Omega}$
- This leads to determinstic ODE's at leading order
- We can choose which processes to include and their probabilities in order to recover the energy cycle equations

Example: Pan and Randall

- We are required to have the following processes:

$E \longrightarrow O$	$A \longrightarrow A$	spontaneous birth (primary initiation)
$E \longrightarrow E$	$A \longrightarrow A+s$	environmental destabilization
$O \longrightarrow E$	$A \longrightarrow A$	death
$O \longrightarrow O$	$A \longrightarrow A-r$	environmental stabilization

- We are required to omit the following processes:

$OO \longrightarrow EO$	$A \longrightarrow A$	competitive exclusion
$OO \longrightarrow OO$	$A \longrightarrow A+2s$	strong stabilization

Example: Pan and Randall

- All other processes are optional:
  - not structurally harmful but complicate the formulae linking the parameters
  - some processes cannot be fully distinguished at the macroscopic level, but only if we consider fluctuations of the system

Example: Yano and Plant

- Main difference is that it excludes:

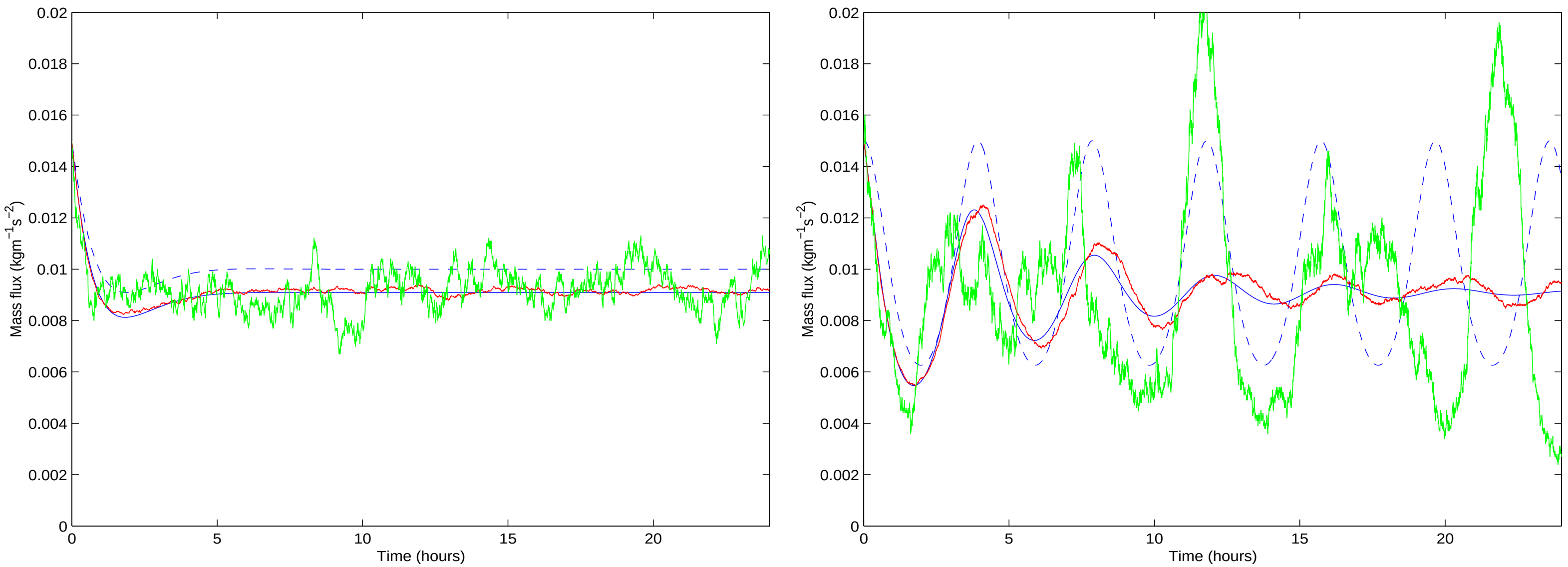
$E \longrightarrow O \qquad A \longrightarrow A$ spontaneous birth (primary initiation)

- and instead requires the process:

$EO \longrightarrow OO \qquad A \longrightarrow A$ induced birth (secondary initiation)

100 realizations for $\Omega = 1000$

Timeseries of M for Pan & Randall (left) and Yano & Plant (right) systems



Dashed blue: solution of ODE. Blue: solution of the ODE derived without assuming $\sigma \ll 1$
Green: a single realization. Red: ensemble mean.

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