A Modelling Framework for Statistical Cumulus Dynamics

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Outline

- Stochastic aspects (a very brief reminder)
- Prognostic aspects
- Combining the stochastic and prognostic
- From the microscopic to the macroscopic
- Some numerical results
- Generalizations?
- Summary

The cumulus ensemble

The Arakawa and Schubert (1974) picture

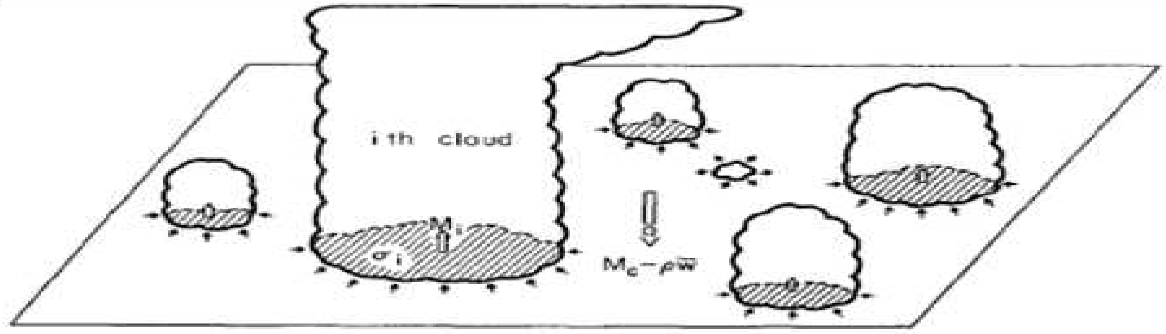


FIG. 1. A unit horizontal area at some level between cloud base and the highest cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud which has lost buoyancy is shown detraining cloud air into the environment.

- Convection characterised by ensemble of convective plumes
- Scale separation in both space and time between cloud-scale and the large-scale environment

Philosophy of this talk

- Convective parameterization can be thought of as an attempt to make a macroscopic (cumulus ensemble) description of a microscopic (plume level) system
  $\Longrightarrow$ we should be interested in techniques that provide firm links between the microscopic and macroscopic
- Such links are a necessary first step towards understanding mesoscopic behaviour (stochastic effects, organization...)
- We are going to see one such technique (a simple one!)
- *Does the Hamiltonian framework provide us with another linking technique?*
- *Is it an appropriate one for further generalization?*

The plumes

- These are characterised by the cloud base mass flux, $M_i = \rho \sigma_i w_i$
- Assume a reasonable plume model exists to compute vertical structure $M_i(z > z_B)$

  But will not ask what exactly the plume model is
- Assume one type of plume only, and so will drop all plume subscripts
- Does not mean that a bulk approximation is needed
- Extension to multiple types is very easy, but would only complicate the presentation

Methodology

- Consider a microscopic-level, individual-based model that evolves according to transition probabilities for births, deaths etc
- We do not know all the rules for such a microscopic model of convection, but they are not just guesswork
- Choose processes and probabilities so that in the macroscopic limit we recover appropriate deterministic ODEs
- We do not know all the rules for a macroscopic model of convection, but they are not just guesswork
- i.e., useful constraints can be found by explicitly calculating the microscopic/macroscopic links

Stochastic aspects of convection

Mass flux variability

- Convective instability is released in discrete events
- The number of clouds in a GCM grid-box is not large enough to produce a steady response to a steady forcing
- In equilibrium, for non-interacting clouds:
  - pdf of mass flux of a single cloud is exponential

    delta function here as only one type
  - number of clouds in finite region is given by Poisson distribution

See previous talk!

Prognostic aspects of convection

Why consider time dependence?

- For relatively rapid forcings, we may wish to consider a prognostic equation for cloud-base mass flux
- Even for steady forcing, it is not obvious
  - that a stable equilibrium must be reached
  - which equilibrium might be reached

Systems for time dependence

- Let A be the vertical integral of in-cloud buoyancy (cloud work function)
- From its definition (after some algebra):

$$\frac{dA}{dt} = F - \gamma M$$

  where A, F and γ are calculable with a plume model
- The convective kinetic energy equation is

$$\frac{dK}{dt} = AM - \frac{K}{\tau_D}$$

- Need further assumption to close these energy equations

Population dynamics system

Wagner and Graf (2011)

- Assume that $K \sim M^p$
- If K equation approaches equilibrium quickly compared to M equation,

$$(p-1)A\frac{dM}{dt} = FM - \gamma M^2$$

- For any $p > 1$, analagous to a Lotka-Volterra system of biological populations competing for resource

Pan and Randall system

- Pan and Randall (1998) choose $p = 2$. i.e.

$$K \sim M^2$$

- Recall $K \sim \sigma w^2$ and $M = \rho\sigma w$ so $p \approx 2$ if variations in w dominate variations in K and M
- Time dependence is a damped oscillator that approaches equilibrium after a few τ_D

Yano and Plant system

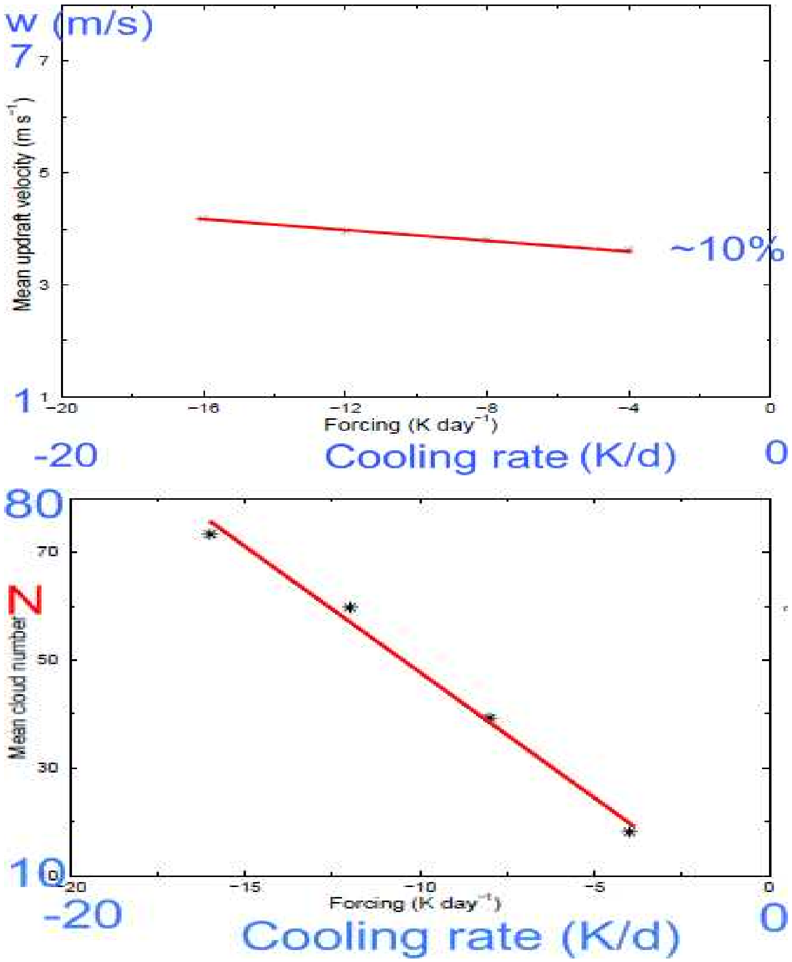
- Yano and Plant (2011) choose $p = 1$. i.e.

$$K \sim M$$

- Recall $K \sim \sigma w^2$ and $M = \rho\sigma w$ so $p \approx 1$ if variations in σ dominate variations in K and M
- This is consistent with scalings and CRM data for changes in mass flux with forcing strength

  Emanuel and Bister 1996; Robe and Emanuel 1996; Grant and Brown 1999; Cohen 2001; Parodi and Emanuel 2009
- Time dependence is periodic orbit about equilibrium state

CRM data for changes in mass flux

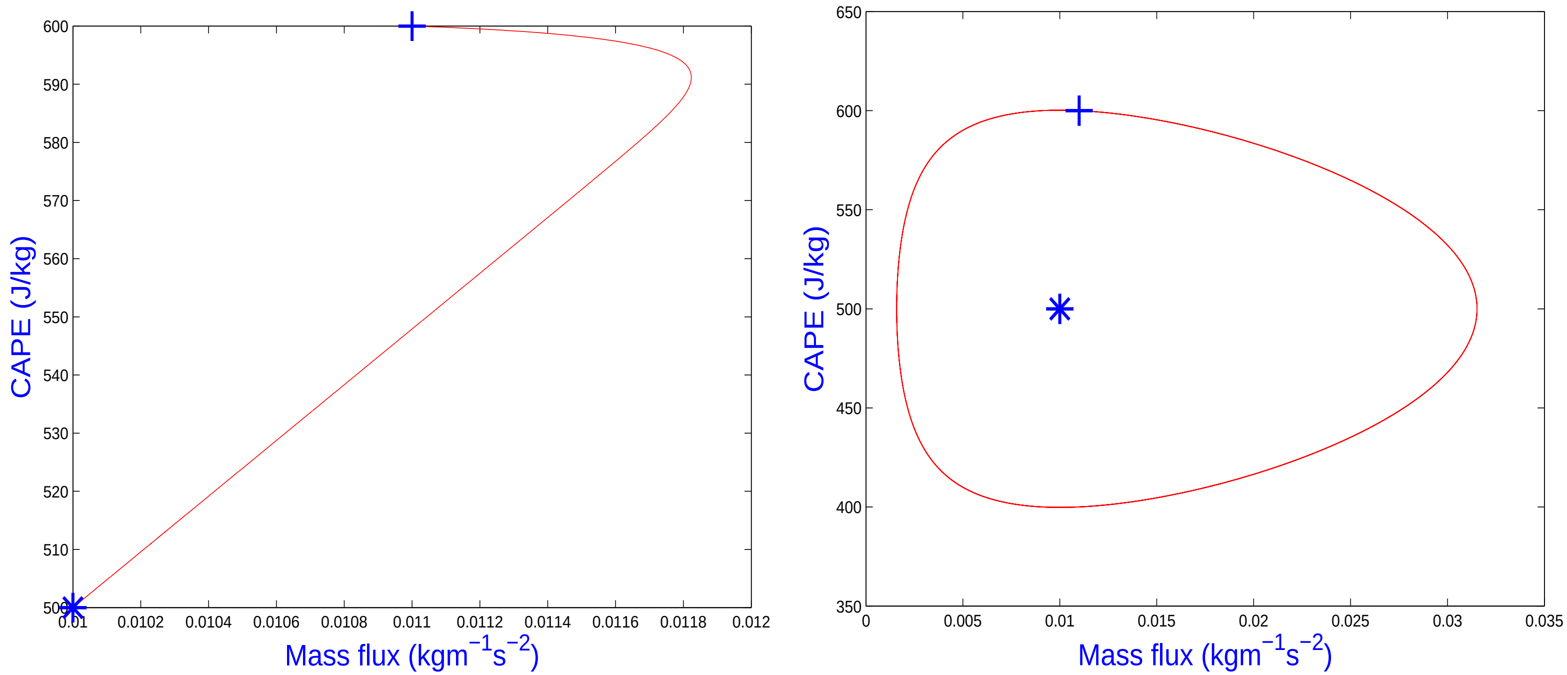


Increased forcing linearly increases mass flux

- achieved by increasing cloud number N (and fractional cloud area σ)
- not the in-cloud velocities
- or the sizes of clouds

Cohen 2001

Illustrative results



Pan & Randall (left) and Yano & Plant (right) systems

$p = 1 + \varepsilon$ has slow spiral into equilibrium;

$p = 1 - \varepsilon$ slow spiral outwards

From the microscopic to the macroscopic

Combined framework

- Develop simple microscopic description for plumes with probabilistic rules
- By choosing appropriate rules can recover...
  - any of the above prognostic systems in the limit of large-system size
  - Poisson distribution of cloud number at equilibrium
- ...if we also take the limit of small cloud fraction, $\sigma \ll 1$

Ingredients

- $P(N, A, \tau)$ is pdf for N clouds and cloud work function A at time τ
- Domain has size Ω elements, each of which may be either empty or occupied by a single cloud
- P evolves through master equation

$$\frac{\partial P(N,A,\tau)}{\partial \tau} = \int dA' \sum T(N,A|N',A')P(N',A',\tau)$$

$$-T(N',A'|N,A)P(N,A,\tau)$$

$T(f|i)$ is probability per unit time of a transition from i to f

Transition Rules

- At each time, look at one site with probability $1-\mu$ or two with probability μ
- Suppose we look at one. With probability $1-(N/\Omega)$ it is empty
- Suppose it is empty:
  - With probability a we allow cloud formation here, $N \longrightarrow N+1$
  - Otherwise it remains empty and atmosphere continues to be destabilized, $A \longrightarrow A+s$
- e.g. for spontaneous birth we have

$$T(N+1,A|N,A') = a(1-\mu)\left(1-\frac{N}{\Omega}\right)\delta(A-A')$$

Possible Processes

$E \xrightarrow{a} O$	$A \rightarrow A$	spontaneous birth (primary initiation)
$E \xrightarrow{1-a} E$	$A \rightarrow A+s$	environmental destabilization
$O \xrightarrow{d} E$	$A \rightarrow A$	death
$O \xrightarrow{1-d} O$	$A \rightarrow A-r$	environmental stabilization
$EO \xrightarrow{b} OO$	$A \rightarrow A$	induced birth (secondary initiation)
$EO \xrightarrow{1-b} EO$	$A \rightarrow A+s-r$	environmental modification
$OO \xrightarrow{c} EO$	$A \rightarrow A$	competitive exclusion
$OO \xrightarrow{1-c} OO$	$A \rightarrow A+2s$	strong stabilization
$EE \xrightarrow{e} EO$	$A \rightarrow A$	birth
$EE \xrightarrow{1-e} EE$	$A \rightarrow A-2r$	strong destabilization

Current status

- We have specified possible rules describing a system of $0 \leq N \leq \Omega$ objects and an environmental field A
- Given the probabilities μ, a, b, c, d, e can integrate this numerically
- To relate this to convection, could allow probabilities to depend on A

  e.g., birth is more likely for larger A
- But how exactly should we choose the appropriate rules to include and appropriate parameters of our system?
- Solution: insist on recovering particular macroscopic systems in the appropriate limits

Recovering the macroscopic systems

System size expansion

- Due to van Kampen, Stochastic processes in physics and chemistry (3rd edn, 2007)
- Widely used in chemistry, biochemistry, population biology...
- Basic idea is to expand master equation in powers of $1/\sqrt{\Omega}$
- Obtain determinstic ODE's at leading order
- Leading correction for a non-infinite system is stochastic and accounts for fluctuations in cloud number via a Fokker-Plank equation
- Will illustrate the method for the spontaneous birth process $E \longrightarrow O$

Decomposition of model variables

- First we introduce a macroscopic timescale

$$t = \Omega^{-1}\tau$$

- For a large system, expect A to be intensive: i.e. almost independent of system size, with some small fluctuations

$$A(t) = \varphi(t) + \Omega^{-1/2}\lambda(t)$$

- Similarly N is extensive

$$N(t) = \Omega\sigma(t) + \Omega^{1/2}\eta(t)$$

so that σ is fraction of domain covered

LHS of master equation

- φ and σ evolve slowly and determinstically whereas λ and η are the fluctuating parts
- Want to capture slow evolution of φ and σ and evolution of the probabilistic behaviour of the fluctuating variables $\Pi(\eta, \lambda, t)$
- The transformation of variables from P to Π gives

$$\frac{\partial P}{\partial \tau} = \Omega^{-1}\left[\frac{\partial \Pi}{\partial t} - \Omega^{1/2}\frac{d\sigma}{dt}\frac{\partial \Pi}{\partial \eta} - \Omega^{1/2}\frac{d\varphi}{dt}\frac{\partial \Pi}{\partial \lambda}\right]$$

RHS of master equation I

- For spontaneous birth, RHS has terms

$$= T(N,A|N-1,A)P(N-1,A,\tau) - T(N+1,A|N,A)P(N,A,\tau)$$

$$= (\Upsilon - 1)T(N+1,A|N,A)P(N,A,\tau)$$

- where we have introduced the transition operator

$$\Upsilon f(N) = f(N-1)$$

- In a large system, transition by one cloud is small effect, and can expand operator as

$$\Upsilon = 1 - \Omega^{-1/2}\frac{\partial}{\partial\eta} + \frac{1}{2}\Omega^{-1}\frac{\partial^2}{\partial\eta^2} \pm \ldots$$

RHS of master equation II

- So terms on RHS of master equation become

$$\left[-\Omega^{-1/2}\frac{\partial}{\partial\eta}+\Omega^{-1}\frac{1}{2}\frac{\partial^2}{\partial\eta^2}+\ldots\right]a(1-\mu)\times$$

$$\frac{1}{\Omega}(\Omega-\Omega\sigma-\Omega^{1/2}\eta)\Pi$$

Macroscopic equation I

- Collecting terms of leading order, $O(1/\Omega)$, we have that

$$\frac{d\sigma}{dt} = a(1-\mu)(1-\sigma)$$

  due to spontaneous birth

- To make contact with existing mass flux models for convection we also take the limit $\sigma \ll 1$

$$\frac{d\sigma}{dt} = a(1-\mu)$$

- Repeating such expansions for all of the possible processes we get

Macroscopic equation II

$$\frac{d\sigma}{dt} = a(1-\mu) + e\mu\sigma\left[2b\mu - d(1-\mu)\right] - c\mu\sigma^2$$

$$\frac{d\varphi}{dt} = \tilde{s}\left[2(1-e)\mu + (1-a)(1-\mu)\right]$$
$$+\sigma\left[2(\tilde{s}-\tilde{r})(1-b)\mu - \tilde{r}(1-d)(1-\mu)\right]$$
$$-2\sigma^2\mu\tilde{r}(1-c)$$

- Recall that $\sigma \propto M$ since there is only one cloud type
- Now just have to choose our processes to get desired structural form of the macroscopic equations
- And automatically get formulae giving microscopic parameters $a, b \ldots$ in terms of macroscopic ones $F, \gamma \ldots$

Example: Pan and Randall

- We are required to have the following processes:

$E \longrightarrow O$	$A \longrightarrow A$	spontaneous birth (primary initiation)
$E \longrightarrow E$	$A \longrightarrow A + s$	environmental destabilization
$O \longrightarrow E$	$A \longrightarrow A$	death
$O \longrightarrow O$	$A \longrightarrow A - r$	environmental stabilization

- We are required to omit the following processes:

$OO \longrightarrow EO$	$A \longrightarrow A$	competitive exclusion
$OO \longrightarrow OO$	$A \longrightarrow A + 2s$	strong stabilization

Example: Pan and Randall

- All other processes are optional:
  - not structurally harmful but complicate the formulae linking the parameters
  - some processes cannot be fully distinguished at the macroscopic level, but only if we consider fluctuations of the system

Example: Yano and Plant

- Main difference is that it excludes:

$E \longrightarrow O \qquad A \longrightarrow A$ spontaneous birth (primary initiation)

- and instead requires the process:

$EO \longrightarrow OO \qquad A \longrightarrow A$ induced birth (secondary initiation)

Example: Population Dynamics

- Only has a macroscopic equation for mass flux, and this requires us to exclude

$E \to O$	$A \to A$	spontaneous birth (primary initiation)

- while including

$EO \to OO$	$A \to A$	induced birth (secondary initiation)
$OO \to EO$	$A \to A$	competitive exclusion

(Actually the microscopic form of this system is already well studied by population biologists: e.g. power spectrum of N has resonance-like peaks)

Some numerical results

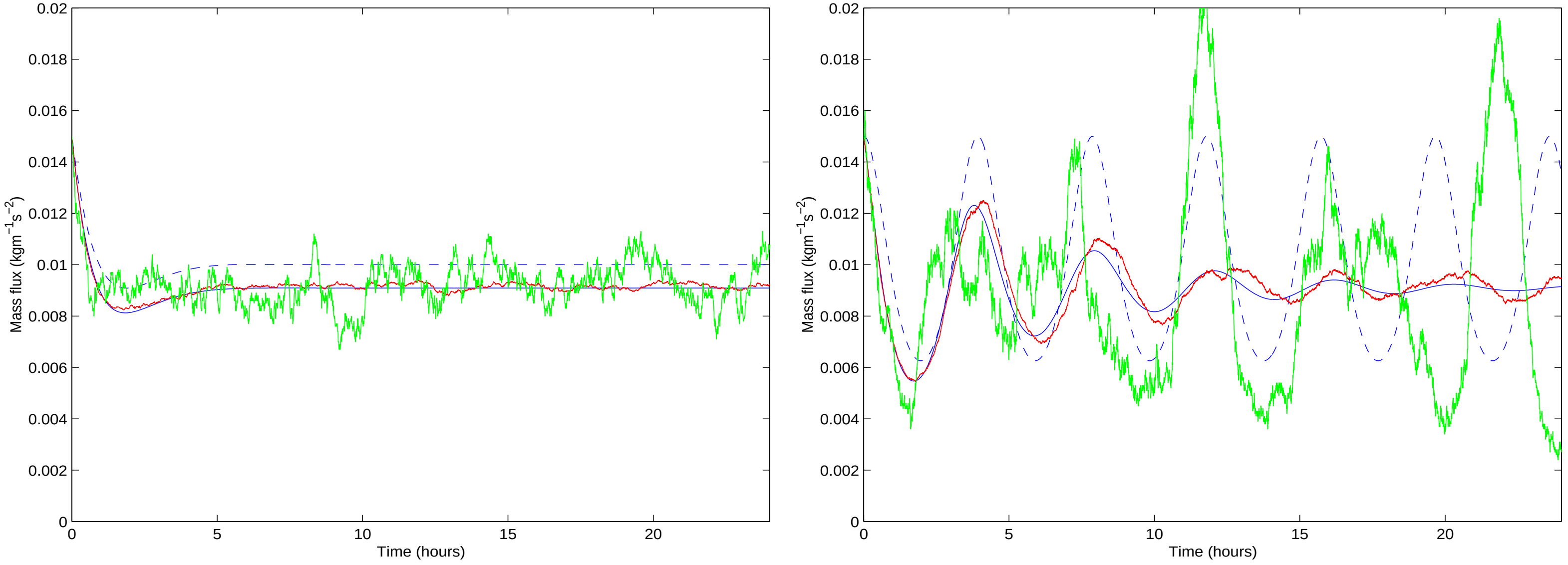
Example results

- Choose some typical values of macroscopic parameters
- Here we show simulation results the minimal microscopic equivalents to the Pan & Randall and Yano & Plant macroscopic systems
- Microscopic parameters are well constrained by our choice of macroscopic parameters

- NB: Have checked that for simulations at different Ω the standard deviations of A and N scale with Ω in just the way assumed in the expansion

100 realizations for $\Omega = 1000$

Timeseries of M for Pan & Randall (left) and Yano & Plant (right) systems

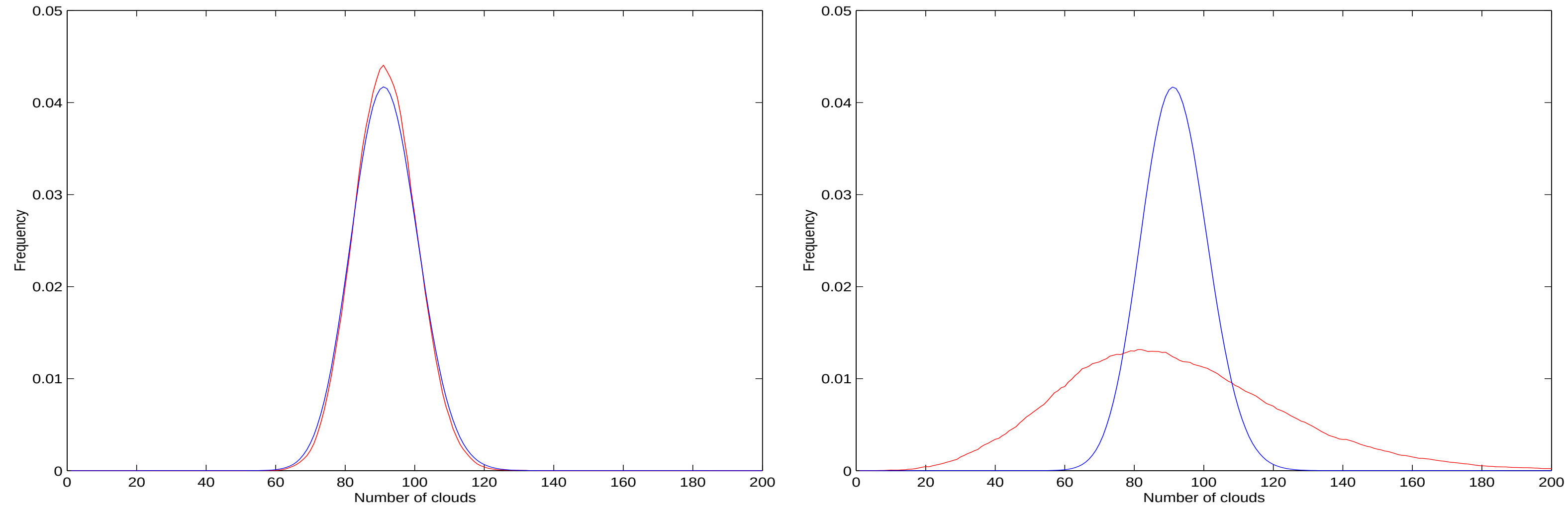


Dashed blue: solution of ODE. Blue: solution of the ODE derived without assuming $\sigma \ll 1$

Green: a single realization. Red: ensemble mean.

Fluctuations in N

pdf of N for Pan & Randall (left) and Yano & Plant (right) systems



Red: model data. Blue: Poisson distribution

Pan & Randall, spontaneous birth depends on number of empty sites, $\Omega - N \approx \Omega$

Yano & Plant, not yet at equilibrium; secondary birth depends on N & A

Generalizations

Generalizations I

- Intermediate models which admit primary and secondary initiation mechanisms
  - would seem more physically reasonable and could very easily be built
- Investigate multiple cloud types
  - Can we can recover the Boltzmann distribution of mass fluxes
  - If so, are there any conditions on $\{F_i, \gamma_{ij}\}$?

Generalizations II

- Investigate stochastic behaviour out of equilibrium
- More generally, might be able to correct CRM data systematically for finite domain effects
- Spatially explicit forms
  - Processes depend on location of site(s), rather than global (nearest neighbour dependencies for transition probabilities)
  - Interactions between patches with rules applied within each patch (less relevant for convection?)
  - Would result in spatial organization

Summary

- Proposed framework for a non-equilibrium, finite N model of cumulus clouds
- Encompasses previous studies in appropriate limits
- Could be a useful intermediate system to study, sitting between CRM/observations and parameterization?
- Many generalizations are possible
- Relationship to Hamiltonian framework (?)