A NON-LOCAL NJL MODEL

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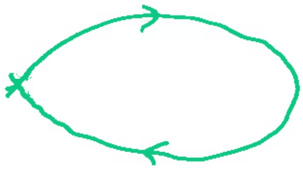
CONSTITUENT QUARKS

CHIRAL SYMMETRY SPONTANEOUSLY BROKEN :-

$$\langle 0 | \bar{\psi}\psi | 0 \rangle \neq 0$$

$$= - \mathrm{Tr} \int \frac{d^4p}{(2\pi)^4} \, i S_F(p)$$

COLOUR, FLAVOUR & DIRAC



MASSLESS QUARKS : $S_F^{-1}(p) \sim \not{p}$

$$\langle \bar{\psi}\psi \rangle \sim -i N_c N_f \int \frac{d^4p}{(2\pi)^4} \frac{\mathrm{tr}\, \not{p}}{p^2} = 0$$

QCD DYNAMICS MUST GENERATE SCALAR CMPT :-

$$S_F^{-1}(p) = \not{p} - m(p)$$

THERE EXISTS A PERTURBATIVE REGION :-

$$m(p^2 \to -\infty) \to 0$$

AT LOW ENERGY, AS 1st APPROX :-

$$m(p^2 \sim 0) \sim \text{CONSTANT} = M$$

M = CONSTANT

* MESONS GENERATED FROM QUARK LOOPS

INTEGRALS ARE UV DIVERGENT:

e.g., $\langle \bar{\psi}\psi \rangle \sim \int d^4p_E \frac{m}{p_E^2 + m^2} \sim \frac{p_E^3 \, dp_E}{p_E^2}$

RESULTS DEPEND ON

REGULARIZATION SCHEME!

* QUARKS NOT CONFINED

⇒ MODELS NOT VALID ABOVE 2m
(~ 600 → 800 MeV)

WHAT ABOUT $\rho(770)$; $a_1(1260)$??

* DOESN'T MATCH ONTO HIGH ENERGY REGIME.

WHICH WOULD BE USEFUL

AT INTERMEDIATE ENERGY.

NJL MODEL

(NAMBU, JONA-LASINIO PR 132, 345)

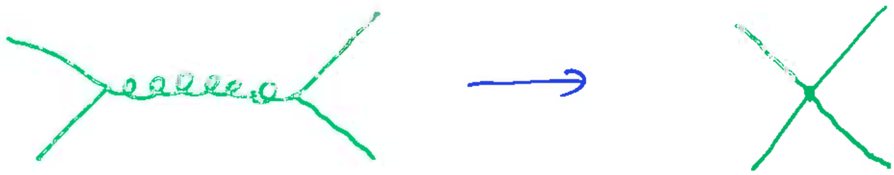
$$S = \int d^4x \; \bar\psi (i\not\partial - m_c)\psi$$
$$+ G_1 \left[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5 \tau^a \psi)^2 \right]$$
$$- G_2 \left[(\bar\psi\gamma^\mu\tau^a\psi)^2 + (\bar\psi\gamma^\mu\gamma_5\tau^a\psi)^2 \right]$$

[ ] TERMS ARE CHIRALLY SYMMETRIC

EXCHANGE OF ONE MASSIVE GLUON

(AT LEADING ORDER IN N_c)

GIVES S_{NJL} WITH $G_2 = \frac{1}{2} G_1$



G_1 AND G_2 ARE $o(N_c^{-1})$

GENERALIZE 4q INTERACTION

BY INTRODUCING NON-LOCALITY

ONE-GLUON EXCHANGE PICTURE:

$$S_{OGE} = \int d^4x \, d^4y \, \bar{\psi}(x) \gamma_\mu \frac{\lambda^a}{2} \psi(x) D^{\mu\nu}(x-y) \bar{\psi}(y) \gamma_\nu \frac{\lambda^b}{2} \psi(y)$$

e.g.: THE GLOBAL COLOUR MODEL

(Cahill & Roberts PR D32, 2419)

$$D^{\mu\nu}(x) = g^{\mu\nu} \delta^{ab} D(x)$$

WITH SOME GUESS AT $D(x)$.

INSTANTON PICTURE

- GLUON FIELD STRENGTH IN $|0\rangle$ CONCENTRATED AROUND x_0
- QUARKS INTERACT BY SCATTERING AT SUCH CENTRES.
- STRENGTH DEPENDS ON SEPARATION OF EACH QUARK FROM x_0.

$$S_{INT} = \int d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4\, \alpha(x_1, x_2, x_3, x_4)$$
$$\left[\bar\psi(x_1)\psi(x_3)\bar\psi(x_2)\psi(x_4) + \bar\psi(x_1)i\gamma_5\tau^a\psi(x_3)\bar\psi(x_2)i\gamma_5\tau^a\psi(x_4)\right]$$
$$+ \beta(\{x\})\left[\bar\psi\gamma^\mu\tau^a\psi\cdot\bar\psi\gamma_\mu\tau^a\psi + \bar\psi\gamma^\mu\gamma_5\tau^a\psi\cdot\bar\psi\gamma_\mu\gamma_5\tau^a\psi\right]$$

USE SEPARABLE INTERACTION :-

$$\tilde\alpha(p_1, p_2, p_3, p_4) = \frac{1}{2}(2\pi)^4 G_1\, f(p_1) f(p_2) f(p_3) f(p_4)\,.\, \delta(p_1 + p_2 - p_3 - p_4)$$

$f(p_i)$: GAUSSIAN

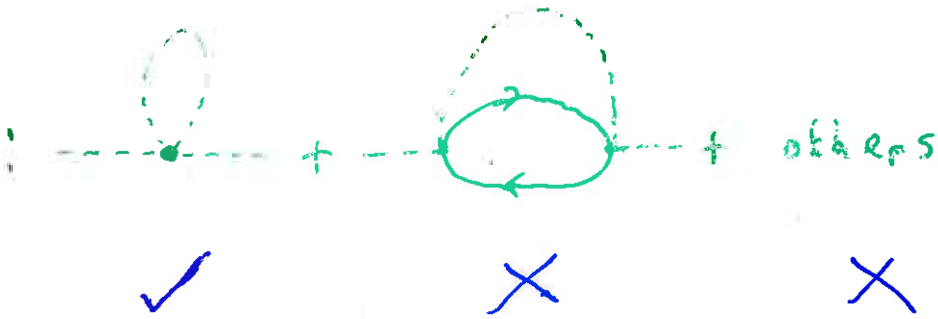
SCHWINGER – DYSON EQN

$$S_F^{-1}(p) = \not p - m_c - \mathrm{Tr}\int \frac{d^4k}{(2\pi)^4}\, S_F(k)\, \Lambda(p,k)$$

LADDER APPROXIMATION :–

(= MEAN FIELD = LO IN N_c^{-1})

Λ :



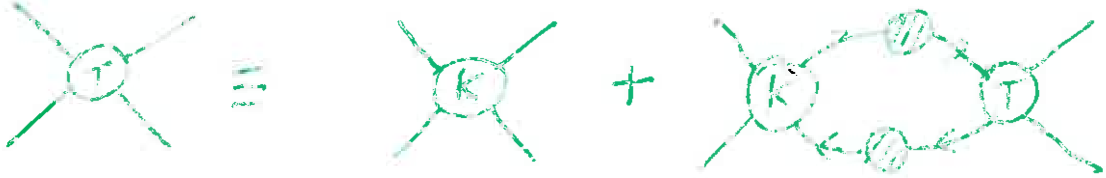
$$M(p) = m_c + 4 G_1 N_c N_f \int \frac{d^4k_E}{(2\pi)^4}\, \frac{M(k_E)}{k_E^2 + M^2(k_E)} \cdot \underbrace{f^2(k_E)\, f^2(p_E)}$$

SEPARABLE FORM DECOUPLES p, k

$$M(p) = m_c + [M(0) - m_c]\, f^2(p)$$

BETHE-SALPETER EQN

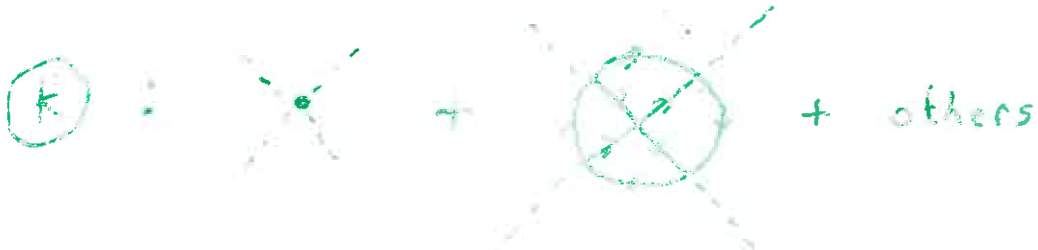
CONSIDER $q\bar{q} \to q\bar{q}$



K IS IRREDUCIBLE

(HAS NO $q\bar{q}$ INTERMEDIATES)

LADDER APPROXIMATION :-



✓ ✗ ✗

WITH SEPARABLE FORM :-

$$T = \prod_i f(p_i)\, T'$$

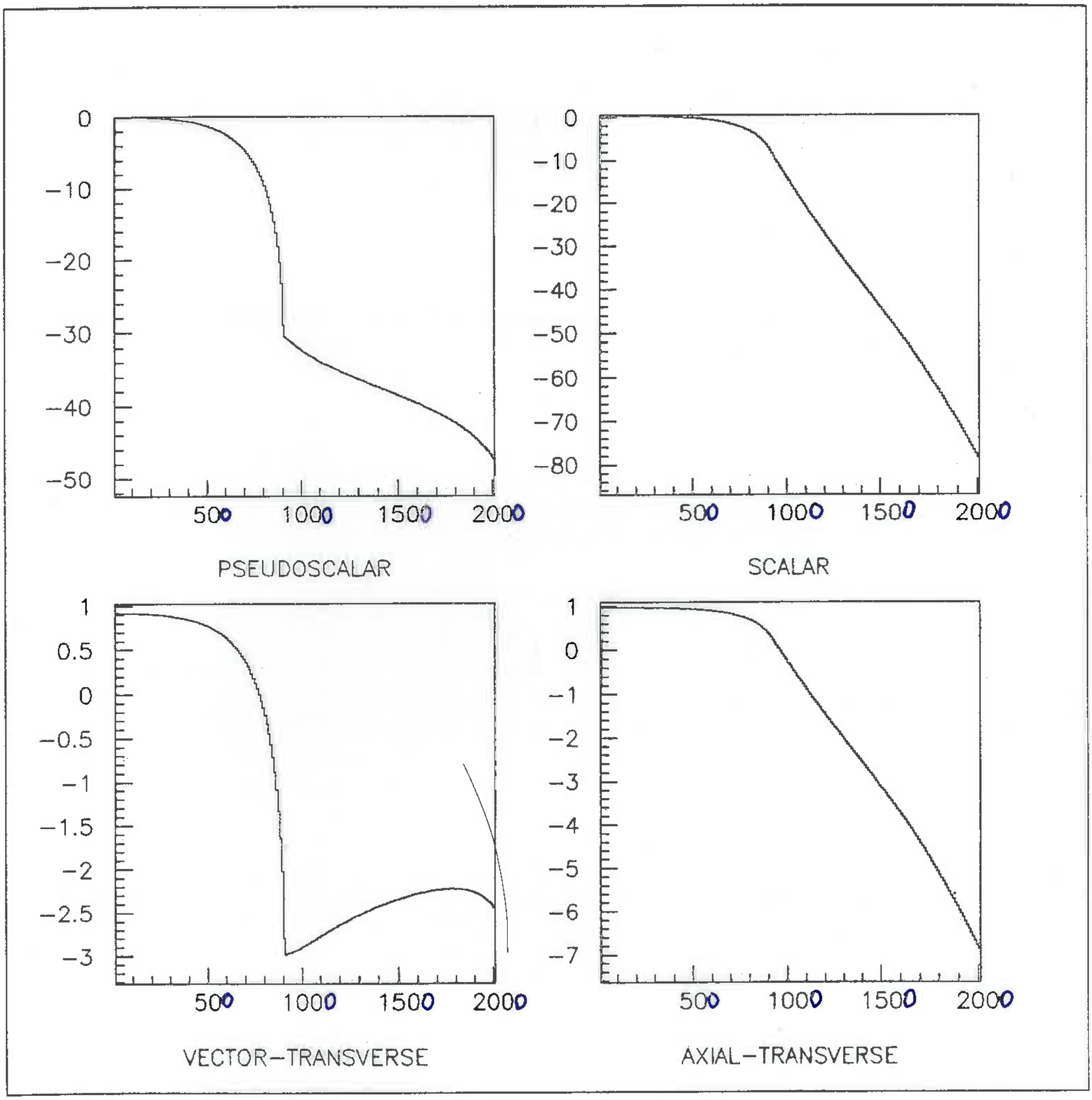
$$T' = G + GJT'$$

WHERE J IS LOOP INTEGRAL 

$$T' = (1 - GJ)^{-1} G$$

MESON POLES AT $\det(1 - GJ) = 0$

DENOMINATORS OF TIME-LIKE PROPOGATORS



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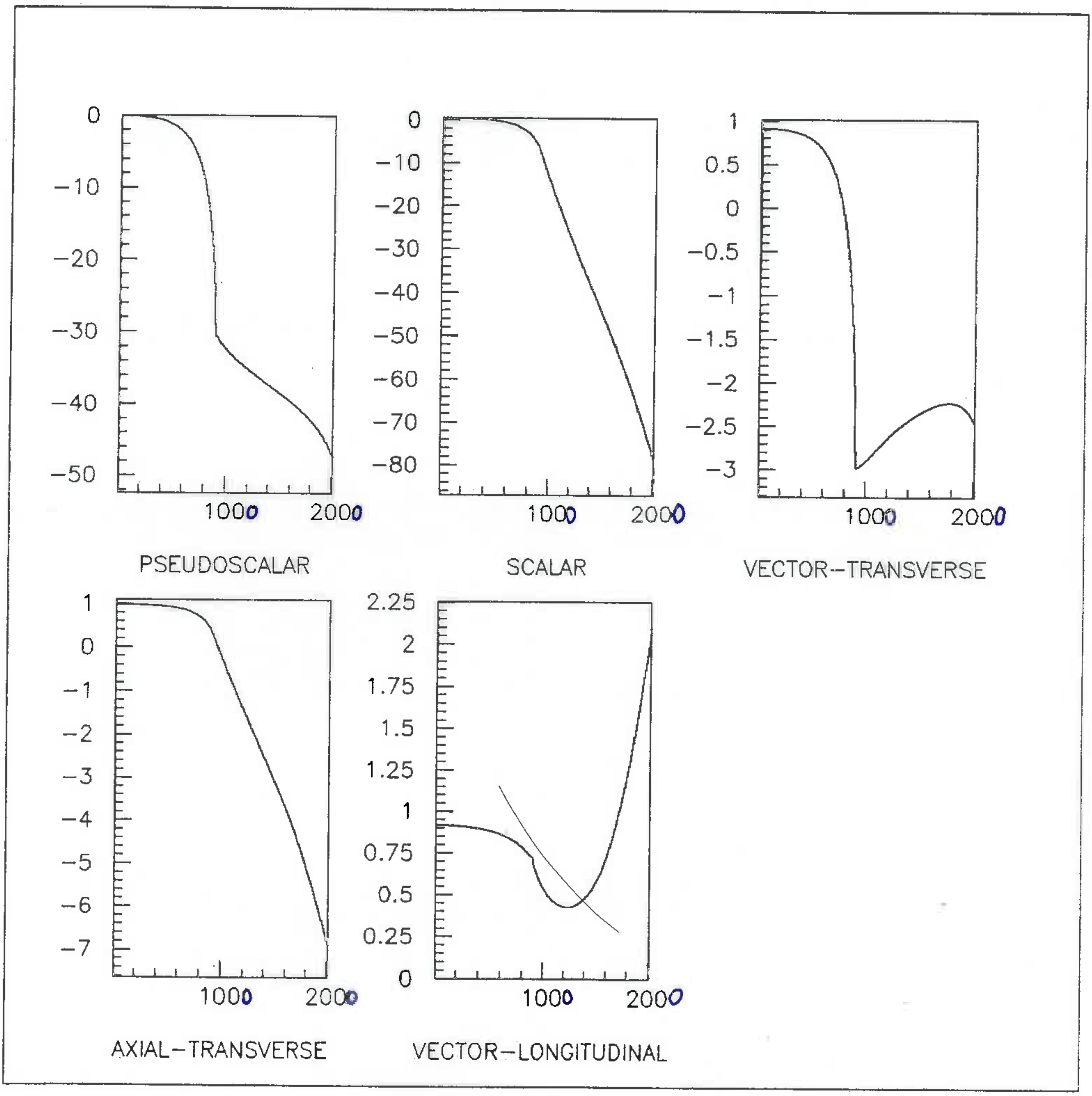
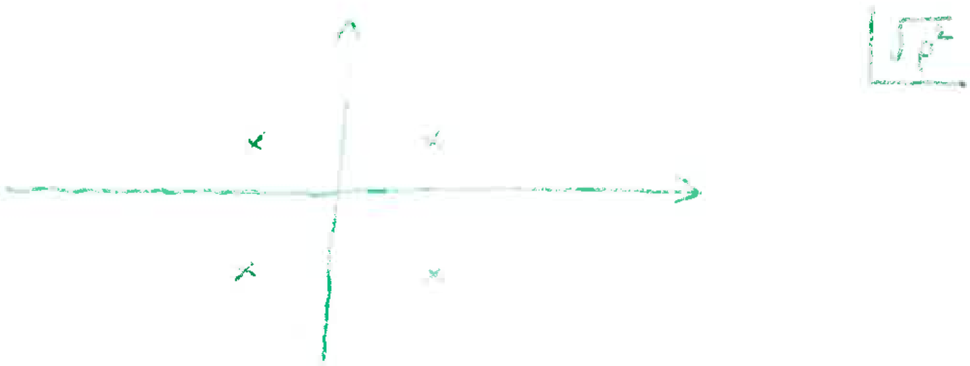


QUARK POLES

$p^2 - m^2(p^2) = 0$ HAS NO SOLNS FOR REAL p^2

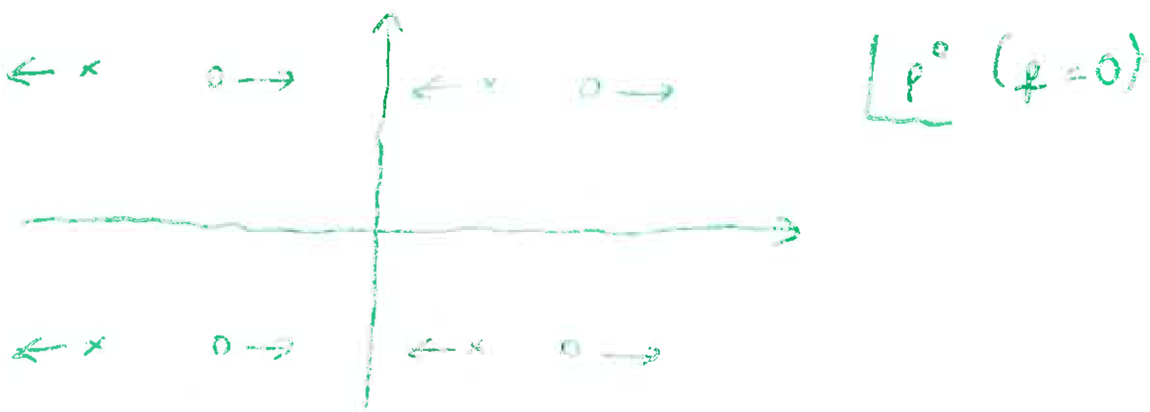
(FOR MOST PARAMETER SETS)



J HAS EXTERNAL TIME-LIKE MMTM:-

$$J = N_c N_f \int \frac{d^4 p_E}{(2\pi)^4} \frac{f^2(p_E + \frac{1}{2} q_E)\, f^2(p_E - \frac{1}{2} q_E) . (\text{trace bit})}{[(p+\frac{1}{2}q)_E^2 + m_+^2][(p-\frac{1}{2}q)_E^2 + m_-^2]}$$

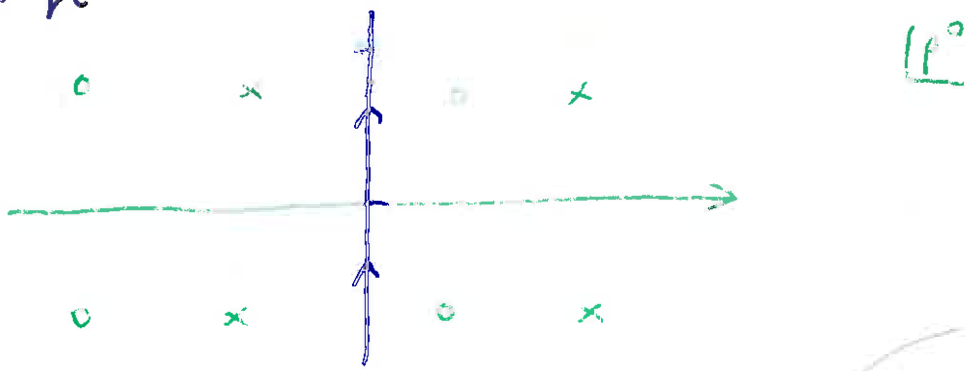
$$q_E = (i\sqrt{-q_E^2}, \underline{0})$$



$\times$: POLE OF $p + \frac{1}{2}q$

$\circ$: POLE OF $p - \frac{1}{2}q$

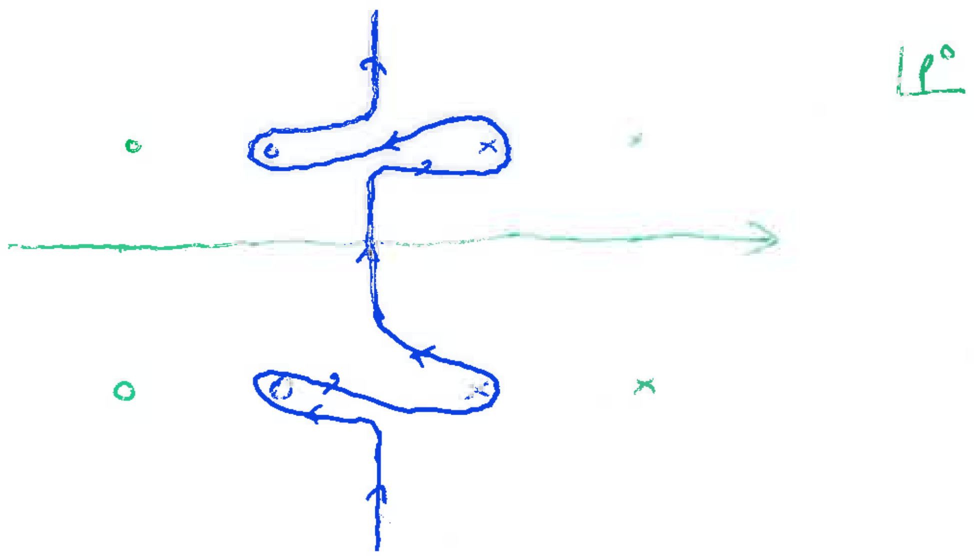
AT $\sqrt{-q_E^2}$ < PINCH :-



— IS INTEGRATION CONTOUR

AT $\sqrt{-q_E^2}$ > PINCH :-

MUST DEFORM CONTOUR TO ENCLOSE APPROPRIATE POLES



ie, INCLUDE RESIDUE CONTRIBUTIONS

(WITHOUT THEM $J \to \infty$)

PION DECAY CONSTANT

$$\langle 0 | A^{\mu a} | \pi^b(q) \rangle = i f_\pi q^\mu \delta^{ab}$$

BUT

$$\partial_\mu \left(\tfrac{1}{2}\bar{\psi}\gamma^\mu\gamma_5\tau^a\psi\right) = i m_c \bar{\psi}\gamma_5\tau^a\psi$$

$$- \int \prod d^4x_i \; \bar{\psi}(x_1) i\gamma_5 \tau^a \psi(x_3) \bar{\psi}(x_2)\psi(x_4) \, \alpha(\Sigma x_i; 3)$$

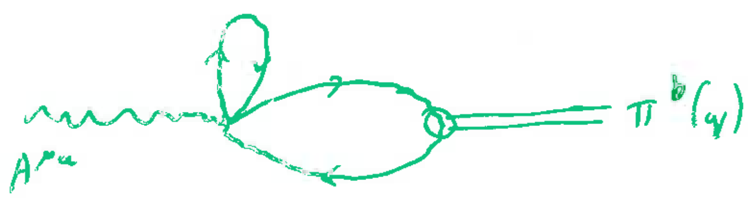
$$\cdot \left[\delta(x-x_1) - \delta(x-x_2) + \delta(x-x_3) - \delta(x-x_4) \right]$$

$+$ similar stuff

TO SATISFY WARD IDENTITIES, NEED EXTRA CONTRIBUTIONS TO $A^{\mu a}$

$$A^{\mu a} = A^{\mu a}_{\text{LOCAL}} + q^\mu (\text{4 QUARK TERMS}) + (\text{TRANS})$$

$\Rightarrow$ 2 MORE DIAGRAMS :-



$\sim$ 30 TO 40% OF f_π

PARAMETER FITTING

4 FREE PARAMETERS : G_1 ; G_2 ; Λ ; m_c

$f(p) = e^{-p_E^2/\Lambda^2}$ $\qquad$ $SU(2)_V$ ASSUMED

FIT TO : $m_\pi = 140$ MeV ; $f_\pi = 93$ MeV ; $m_\rho = 770$ MeV

LEAVING ONE UNDETERMINED
(TAKEN TO BE $m(0)$)

FIND THAT : $m(0) \sim 310 \rightarrow 370$ MeV

$m_c \sim 8 \rightarrow 11$ MeV

$\Lambda \sim 1050 \rightarrow 850$ MeV

MESON MASSES

'σ' (CORRELATED 2π EXCHANGE)

$435 \rightarrow 465$ MeV

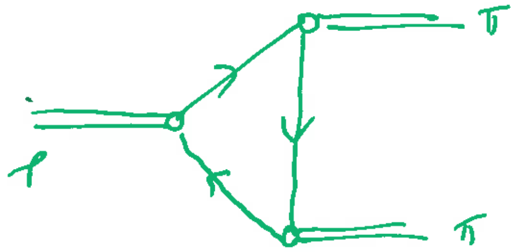
$a_1(1260)$: $950 \rightarrow 1060$ MeV

EXPERIMENT* : 1230 ± 40 MeV

(* : STILL A BIT CONTROVERSIAL)

3 MESON VERTICES

EVALUATE DIAGRAMS LIKE: –



AT $m(0) = 325$

	PREDICTION	EXPERIMENT
$\Gamma(\rho \to \pi\pi)$	128	151
$\Gamma(a_1 \to \rho\pi)$	572	~ 400
$a_1 \to \rho\pi$ D WAVE CONTRIBUTION	8%	'small'
$\Gamma(a_1 \to \sigma\pi)$	260	—
$\Gamma(\sigma \to \pi\pi)$	5.7	—

fff VERTEX FORM FACTORS