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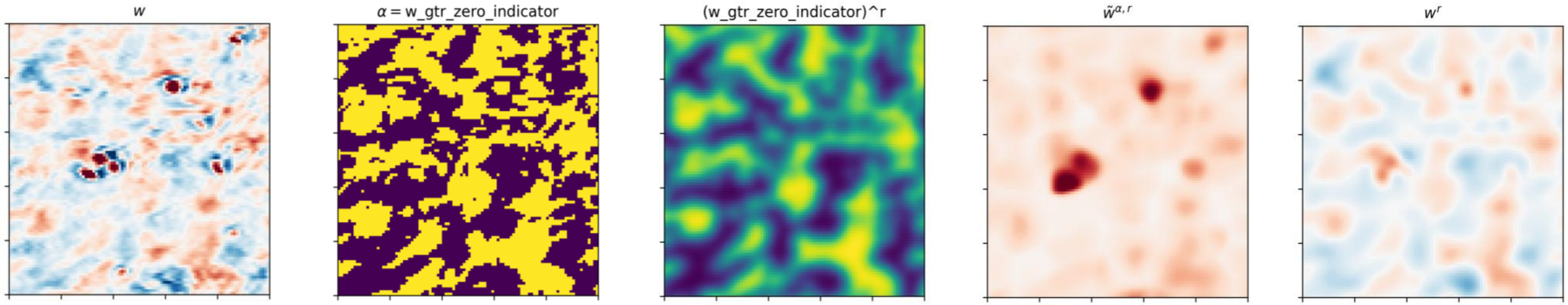
Examining EDMF-type approaches in the grey zone using conditional filtering



***Dan Shipley**, Peter Clark, Bob Plant*

Workshop on Navigating the Turbulence Grey Zone in Numerical Weather Prediction

University of Exeter

25/06/2025

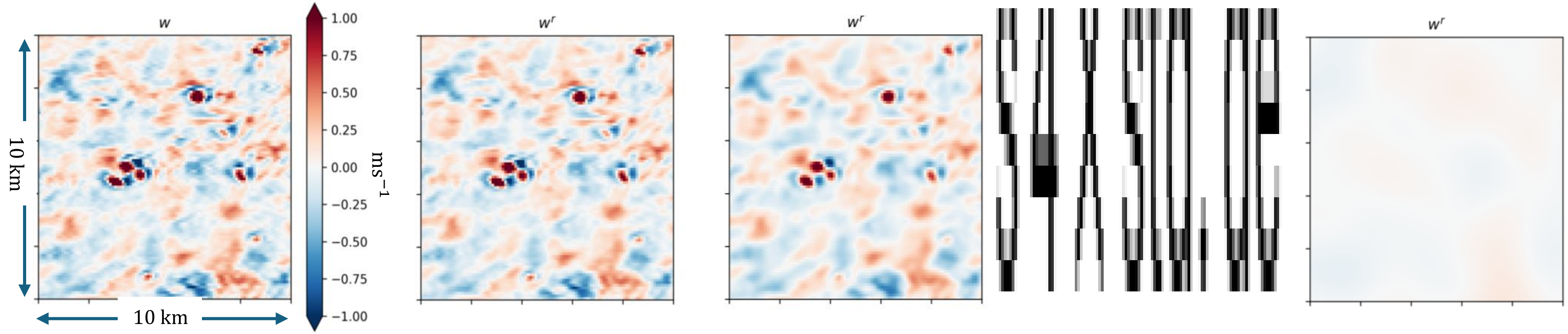
Motivation

- Mass flux (& eddy diffusivity-mass flux) parametrizations of moist convection are ubiquitous in models with grid lengths > O(10) km.
- Models run with grid lengths < O(10) km – entering the grey zone of deep convection – often perform better without such a convection scheme, instead using LES-type closures for mixing alongside a 1D BL scheme (e.g. UM RAL3).
- There is no a priori reason for this – work is ongoing to modify MF-type schemes for use in such models (e.g. CoMorph trailblazer).
- At the other end of the grey zone: LES-type closures are being made more sophisticated (e.g. higher moment closures like 3DTE, dynamic methods etc.) to perform better at coarser resolution.
- We would like these approaches to meet in the middle!
    - Smooth *and physically consistent* transition of behaviour from resolved to subfilter across processes and across scales.
    - Requires estimation of *length scales* to know where you are within the grey zone.
    - Requires a transition from 1D to 3D.
- To do so, we need a way to analyse both approaches within the same formalism: this is provided by **conditional filtering**.

Spatial filtering (recap for notation)

- We care about the transport and evolution of physical variables φ:
  - $\frac{\partial \varphi}{\partial t} + \nabla \cdot \mathbf{u}\varphi = S_\varphi$ (1)
- Since we cannot resolve all scales of motion, we need to average the governing equations.
  - In our case this is ***integral spatial filtering*** with characteristic ***filter length scale*** ℓ_f.
  - Applying the filter to the variable φ gives the ***resolved variable*** φ^r.

Examples: Gaussian filters applied* to vertical velocity from BOMEX ($\Delta x = 100$ m) at $z = 900$ m



Raw model field
($\Delta x = 100$ m)

$\ell_\mathrm{f} = 200\mathrm{m}^{**}$
$(= 2\Delta x)$

$\ell_\mathrm{f} = 400\mathrm{m}$
$(= 4\Delta x)$

$\ell_\mathrm{f} = 1000\mathrm{m}$
$(= 10\Delta x)$

$\ell_\mathrm{f} = 4000\mathrm{m}$
$(= 40\Delta x)$

** ℓ_f approximated as 4 × std. dev. of Gaussian kernel

* Using the subfilter Python package, https://github.com/ReadingClouds/Subfilter (thanks to Peter Clark & Todd Jones)

Spatial filtering (recap for notation)

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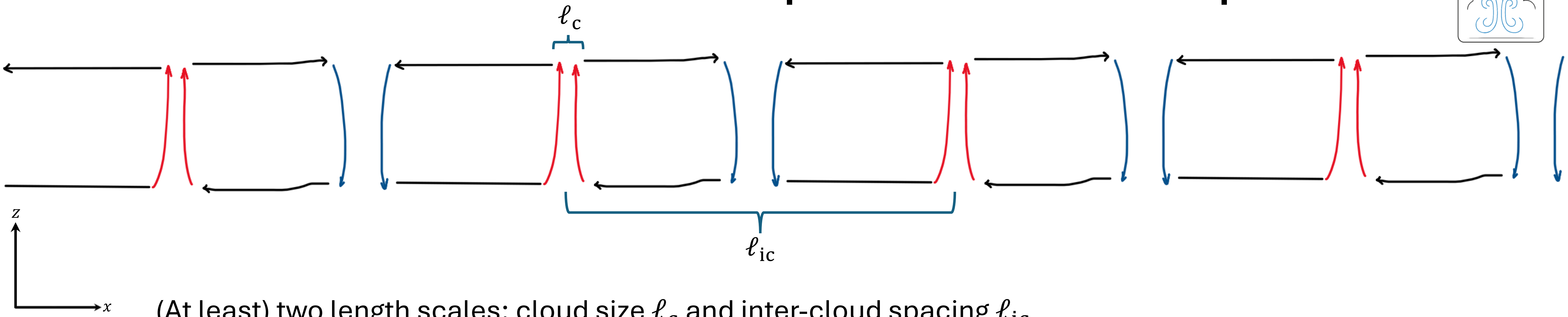
  - $\frac{\partial \varphi}{\partial t} + \nabla \cdot \mathbf{u}\varphi = S_\varphi$ (1)
- Since we cannot resolve all scales of motion, we need to average the governing equations.
  - In our case this is ***integral spatial filtering*** with characteristic ***filter length scale*** ℓ_f.
  - Applying the filter to the variable φ gives the ***resolved variable*** φ^r.
- Filtering the governing equation (1) gives an equation for the evolution of the variable φ^r resolved on scale ℓ_f:
  - $\frac{\partial \varphi^\mathrm{r}}{\partial t} + \nabla \cdot \mathbf{u}^\mathrm{r}\varphi^\mathrm{r} = S_\varphi^r - \nabla \cdot s(\mathbf{u}, \varphi)$, $\quad s(\mathbf{u}, \varphi) \coloneqq (\mathbf{u}\varphi)^\mathrm{r} - \mathbf{u}^\mathrm{r}\varphi^\mathrm{r}$ (subfilter flux of φ)
  - $s(\mathbf{u}, \varphi)$ will in general depend on both the flow and the filter length scale ℓ_f.

It is this term that we must model in convection parametrization!

Sketch of the convection parametrization problem



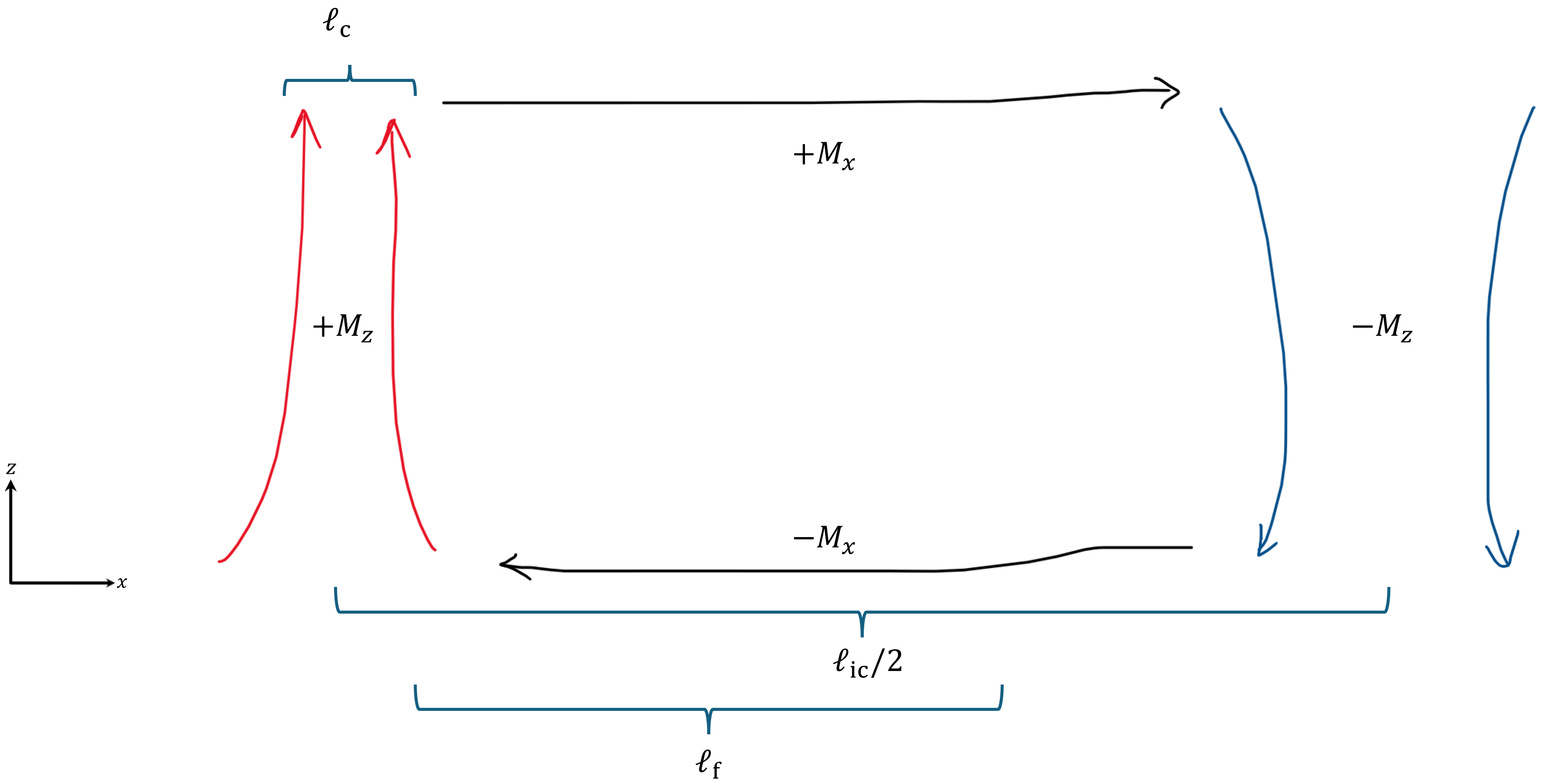
(At least) two length scales: cloud size ℓ_c and inter-cloud spacing ℓ_{ic}

Two “nice” regimes:

- $\ell_f \gg \ell_{ic}$: fully parametrized; mass flux/RANS closures apply (i.e. 1D, largely time-independent)
- $\ell_f \ll \ell_c$: fully resolved; LES closures apply (i.e. 3D, time-dependent; well within inertial sub-range)

In between is the grey zone where neither set of assumptions is valid.

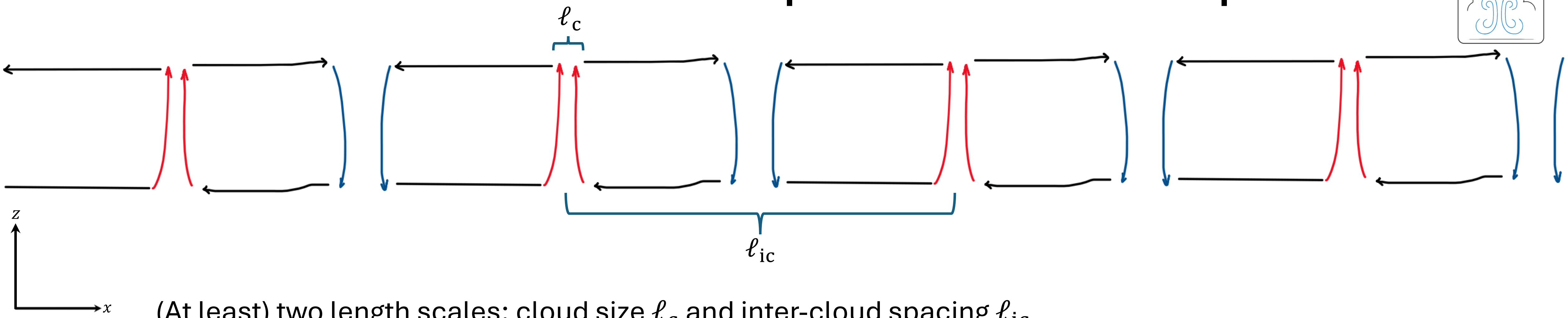
Especially difficult is the region where $\ell_c < \ell_f \lesssim \ell_{ic}$:



For $\ell_\mathrm{c} < \ell_\mathrm{f} \lesssim \ell_\mathrm{ic}$, the filter scale is embedded inside a single overturning circulation – so horizontal fluxes *must* become important!
Thus if not before, a scheme must transition from 1D -> 3D in this regime.

Therefore the scheme must be able to estimate $\ell_\mathrm{ic}$and ℓ_c!

Sketch of the convection parametrization problem



(At least) two length scales: cloud size ℓ_{c} and inter-cloud spacing ℓ_{ic}

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- $\ell_{\mathrm{f}} \gg \ell_{\mathrm{ic}}$: fully parametrized; mass flux/RANS closures apply (i.e. 1D, largely time-independent)
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In between is the grey zone where neither set of assumptions is valid.
Especially difficult is the region where $\ell_{\mathrm{c}} < \ell_{\mathrm{f}} \lesssim \ell_{\mathrm{ic}}$.

How can a mass flux convection scheme be made scale-aware in such a way that the hand-over to both explicitly-resolved convection, and 3D turbulence-parametrized convection, is smooth and physically consistent?

Conditional filtering

- Introduce “indicator functions” based on physical conditions (i.e. a set of masks for e.g. q_cl > threshold, or buoyancy flux > 0 etc.):

$$I_i = \begin{cases} 1 & \text{where condition } i \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

- Define a conditional spatial filter by multiplying a field by I_i, then filtering:

$$\sigma_i := I_i^{\mathrm{r}}, \qquad \sigma_i \varphi_i^{\mathrm{r}} := (I_i \varphi)^{\mathrm{r}}$$

Generalisation of mass flux (Thuburn et al. 2018; also Yano 2014, and others as far back as Dopazo 1977)

Conditional filtering

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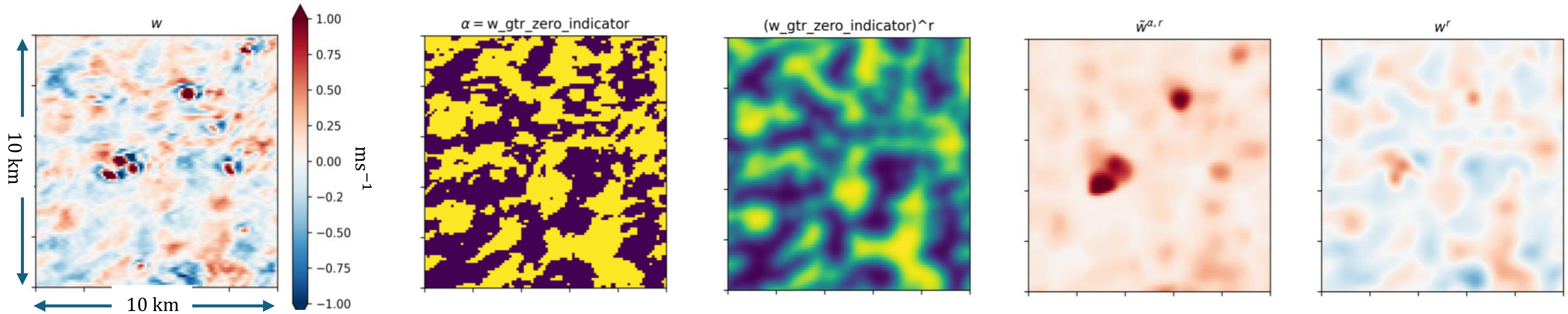
Generalisation of mass flux (Thuburn et al. 2018; also Yano 2014, and others as far back as Dopazo 1977)

***Examples:* $\ell_{\mathrm{f}} = 1000\ \mathrm{m}$ *conditional Gaussian filter applied* to w from BOMEX (Δ*$x = 100\ \mathrm{m}$*) at* $z = 900\ \mathrm{m}$**



Raw model field ($\Delta x = 100\ \mathrm{m}$)

$$I_{w>0} = \begin{cases} 1 & \text{where } w > 0 \\ 0 & \text{otherwise} \end{cases}$$

$\sigma_{w>0} \coloneqq I_{w>0}^{\mathrm{r}}$

$w_{w>0}^{\mathrm{r}} \coloneqq (I_{w>0} w)^{\mathrm{r}}$

w^{r}

* Built on top of the subfilter Python package; not yet part of public code.

Conditional filtering

- Introduce "indicator functions" based on physical conditions (i.e. a set of masks for e.g. q_cl > threshold, or buoyancy flux > 0 etc.):

$$I_i = \begin{cases} 1 & \text{where condition } i \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

- Define a conditional spatial filter by multiplying a field by I_i, then filtering:

$$\sigma_i \coloneqq I_i^{\mathrm{r}}, \qquad \sigma_i \varphi_i^{\mathrm{r}} \coloneqq (I_i \varphi)^{\mathrm{r}}$$

- Higher-order quantities can then be defined analogously to normal spatial filtering:

$$s_i(\mathbf{u}, \varphi) \coloneqq (\mathbf{u}\varphi)_i^{\mathrm{r}} - \mathbf{u}_i^{\mathrm{r}} \varphi_i^{\mathrm{r}}$$

- Then the subfilter flux of φ, $s(\mathbf{u}, \varphi)$ – ***the quantity we're trying to model!*** – can be written *exactly* in terms of conditionally-filtered quantities (e.g. Siebesma 1995):

$$s(\mathbf{u}, \varphi) = \sum_i \sigma_i (\mathbf{u}_i^{\mathrm{r}} - \mathbf{u}^{\mathrm{r}})(\varphi_i^{\mathrm{r}} - \varphi^{\mathrm{r}}) + \sum_i \sigma_i s_i(\mathbf{u}, \varphi)$$

"coherent structures"
(i.e. "mass flux")

"incoherent turbulence"
(or just "not coherent convection")

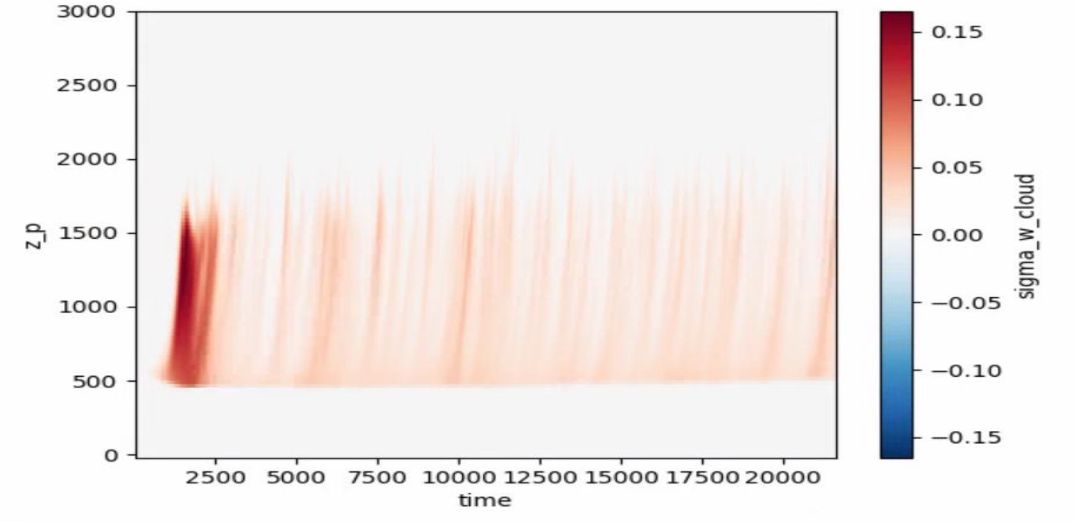
Generalisation of mass flux (Thuburn et al. 2018; also Yano 2014, and others as far back as Dopazo 1977)

This identity (and extensions to higher moments) can be used to exactly relate budgets of e.g. TKE, buoyancy variance etc. term-by-term to conditionally-filtered quantities (a future talk!)

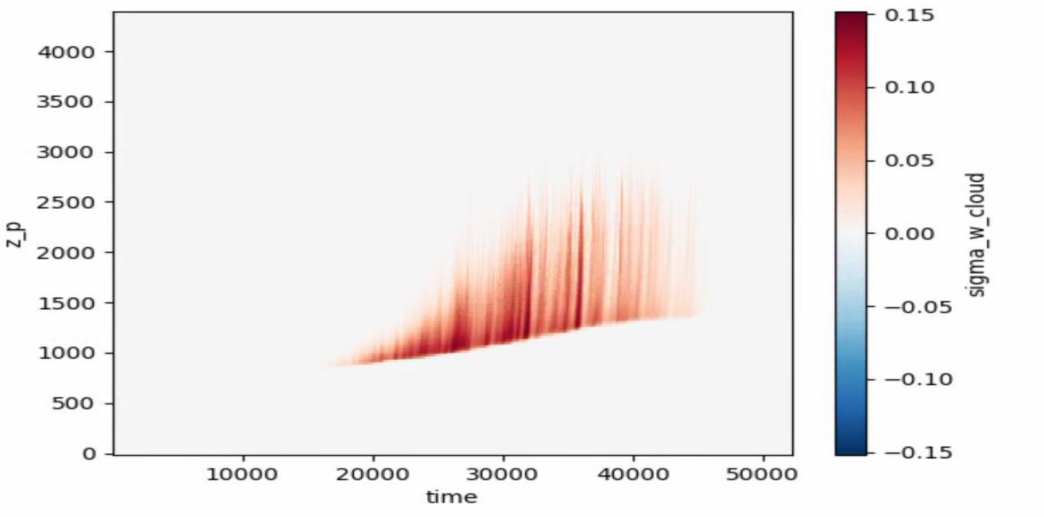
Conditional filtering: methodology

- For this talk, choose buoyancy flux-based conditions:
    - Condition 1 (buoyant updraft): $wb > 0 \text{ AND } w > 0$
    - Condition 2 (negatively buoyant downdraft): $wb > 0 \text{ AND } w < 0$
    - Condition 3 ("environment"): $wb < 0$
- Have also tested w-only conditions and q_cl-dependent conditions (not shown)
- Apply conditional filtering to MONC LES of:

BOMEX:



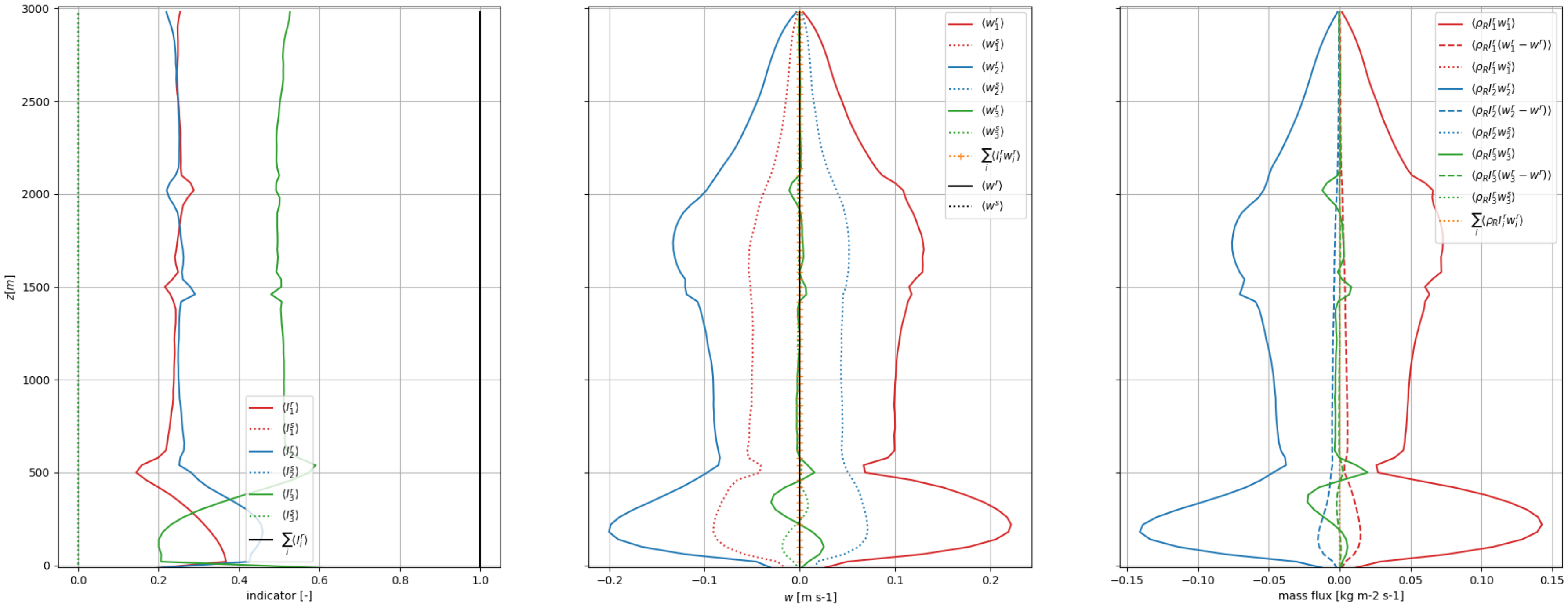
ARM:



Conditional filtering: area fraction & mass flux

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_{\mathrm{f}} = 200\ \mathrm{m}\ (= 2\Delta x)$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$



Domain-averaged: $\sigma_i = I_i^{\mathrm{r}}$ $w_i^{\mathrm{r}} = (I_i w)^{\mathrm{r}} / \sigma_i$ $\rho_0 \sigma_i w_i^{\mathrm{r}} = \rho_0 (I_i w)^{\mathrm{r}}$ (solid lines)

$\rho_0 \sigma_i (w_i^{\mathrm{r}} - w^{\mathrm{r}})$ (dashed lines)

Area fraction

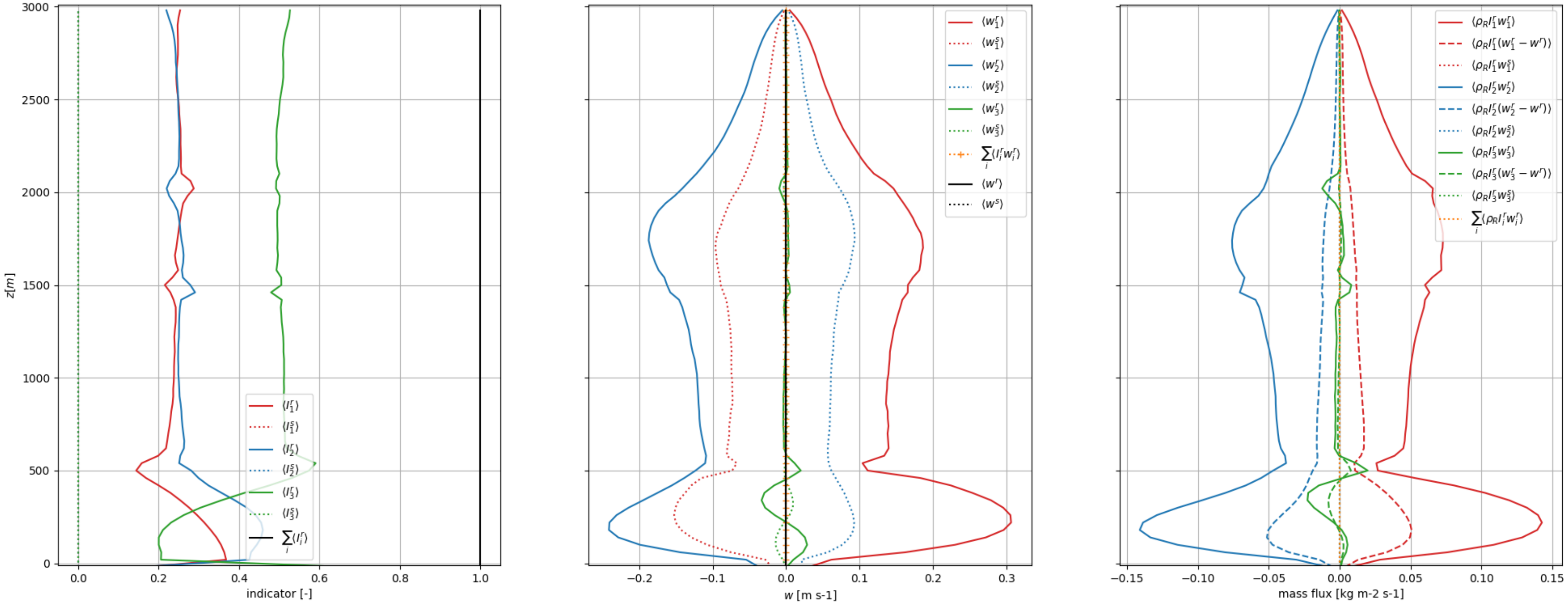
Conditionally-resolved vertical velocity

Mass flux

Conditional filtering: area fraction & mass flux

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} = 400\ \mathrm{m}\ (= 4\Delta x)$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$



Domain-averaged:

$\sigma_i = I_i^\mathrm{r}$

$w_i^\mathrm{r} = (I_i w)^\mathrm{r} / \sigma_i$

$\rho_0 \sigma_i w_i^\mathrm{r} = \rho_0 (I_i w)^\mathrm{r}$ (solid lines)
$\rho_0 \sigma_i (w_i^\mathrm{r} - w^\mathrm{r})$ (dashed lines)

Area fraction

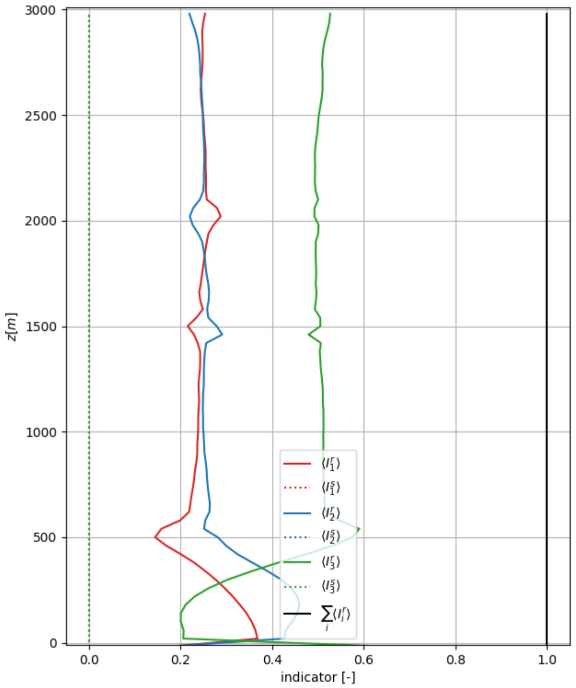
Conditionally-resolved vertical velocity

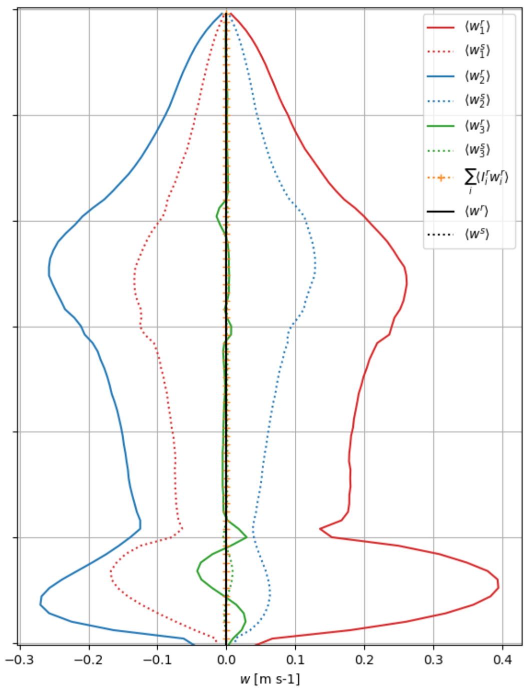
Mass flux

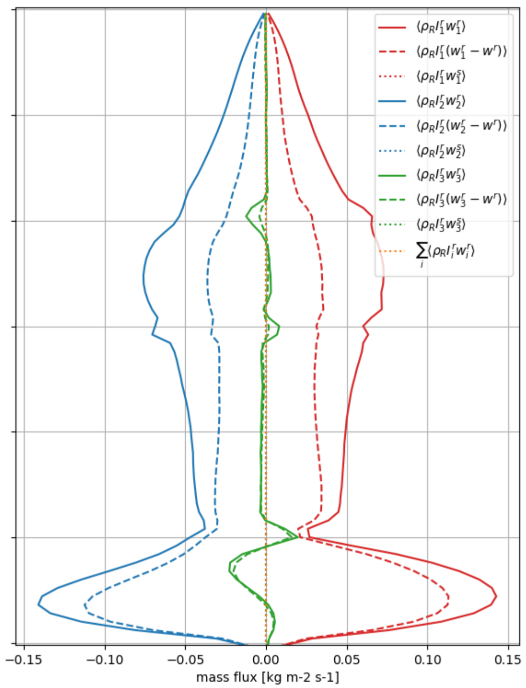
Conditional filtering: area fraction & mass flux

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_{\mathrm{f}} = 1000\ \mathrm{m}\ (= 10\Delta x)$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$







Domain-averaged: $\sigma_i = I_i^{\mathrm{r}}$ $\quad w_i^{\mathrm{r}} = (I_i w)^{\mathrm{r}} / \sigma_i$ $\quad \rho_0 \sigma_i w_i^{\mathrm{r}} = \rho_0 (I_i w)^{\mathrm{r}}$ (solid lines)

$\rho_0 \sigma_i (w_i^{\mathrm{r}} - w^{\mathrm{r}})$ (dashed lines)

Area fraction

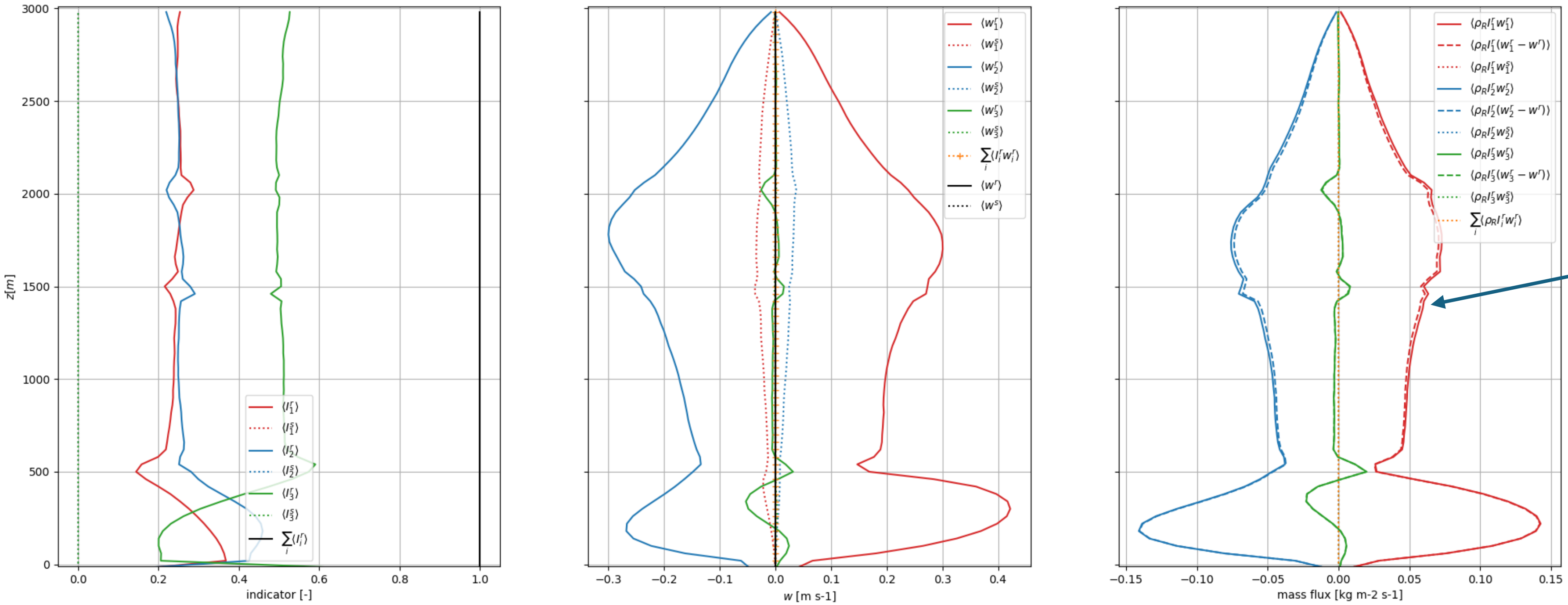
Conditionally-resolved vertical velocity

Mass flux

Conditional filtering: area fraction & mass flux

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} = 4000\ \mathrm{m}\ (= 40\Delta x)$

Condition 1:
$wb > 0 \text{ AND } w > 0$
Condition 2:
$wb > 0 \text{ AND } w < 0$
Condition 3:
$wb < 0$



$\rho_0\sigma_i(w_i^\mathrm{r} - w^\mathrm{r})$
(mass flux relative to resolved flow) saturates to standard mass flux, i.e. resolved circulation (w^r) is negligible
$\Rightarrow$ Filter scale now > inter-cloud spacing

$\rho_0\sigma_i w_i^\mathrm{r} = \rho_0(I_i w)^\mathrm{r}$ (solid lines)
$\rho_0\sigma_i(w_i^\mathrm{r} - w^\mathrm{r})$ (dashed lines)

Note: domain-averaged σ_i and $\rho_0\sigma_i w_i^r$ are invariant with filter scale, but w_i^r and $\rho_0\sigma_i(w_i^r - w^r)$ (the CoMorph mass flux) are not.
$\Rightarrow$ Invariant quantities give "sanity checks", but we also need to get the right dependence of $\rho_0\sigma_i(w_i^r - w^r)$ with scale

vertical velocity

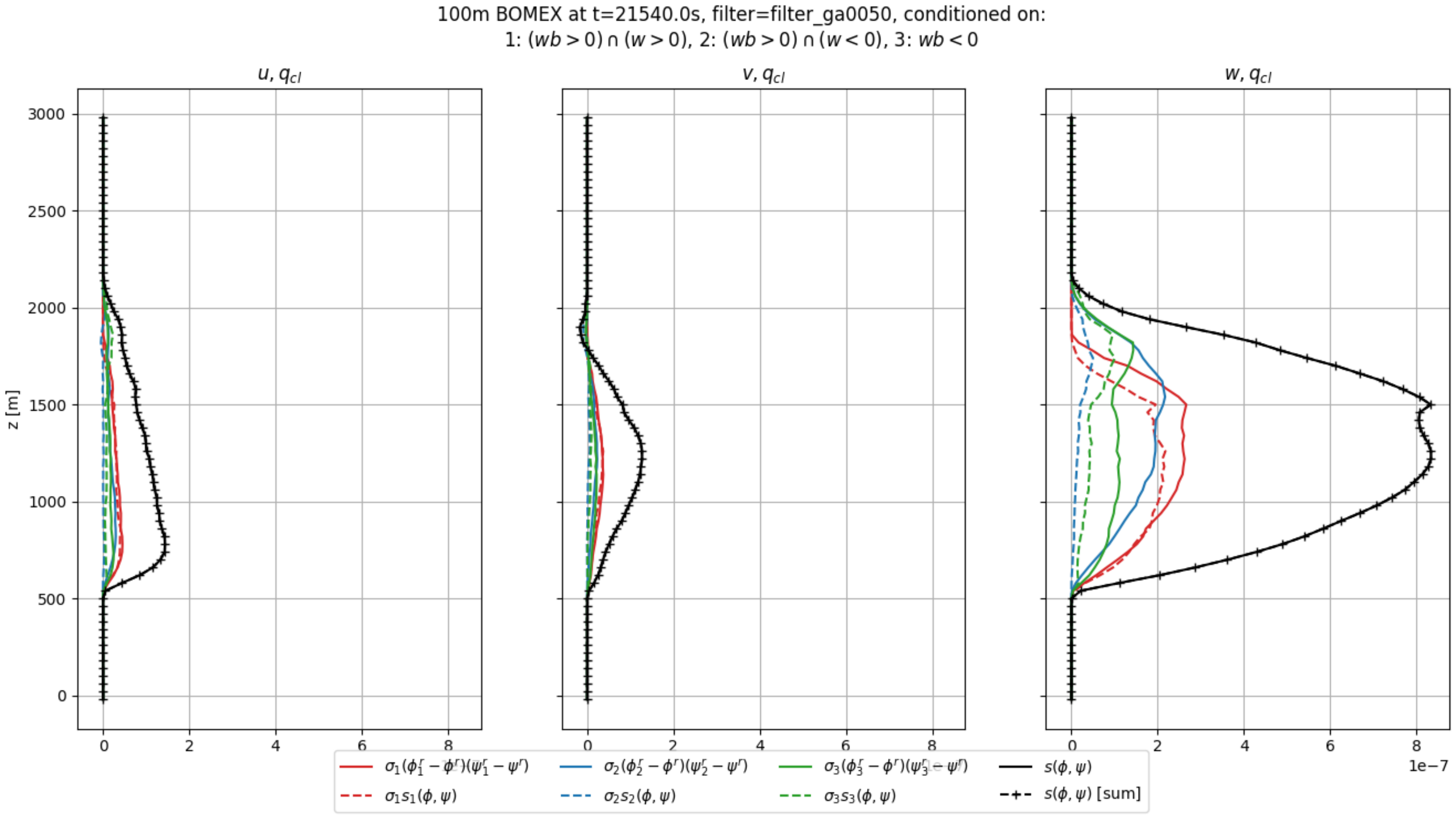
Mass flux

Conditional filtering: vertical fluxes

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_f = 200\ \mathrm{m}\ (= 2\Delta x)$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

— total
— coherent
- - incoherent
-+- total
(Siebesma identity)



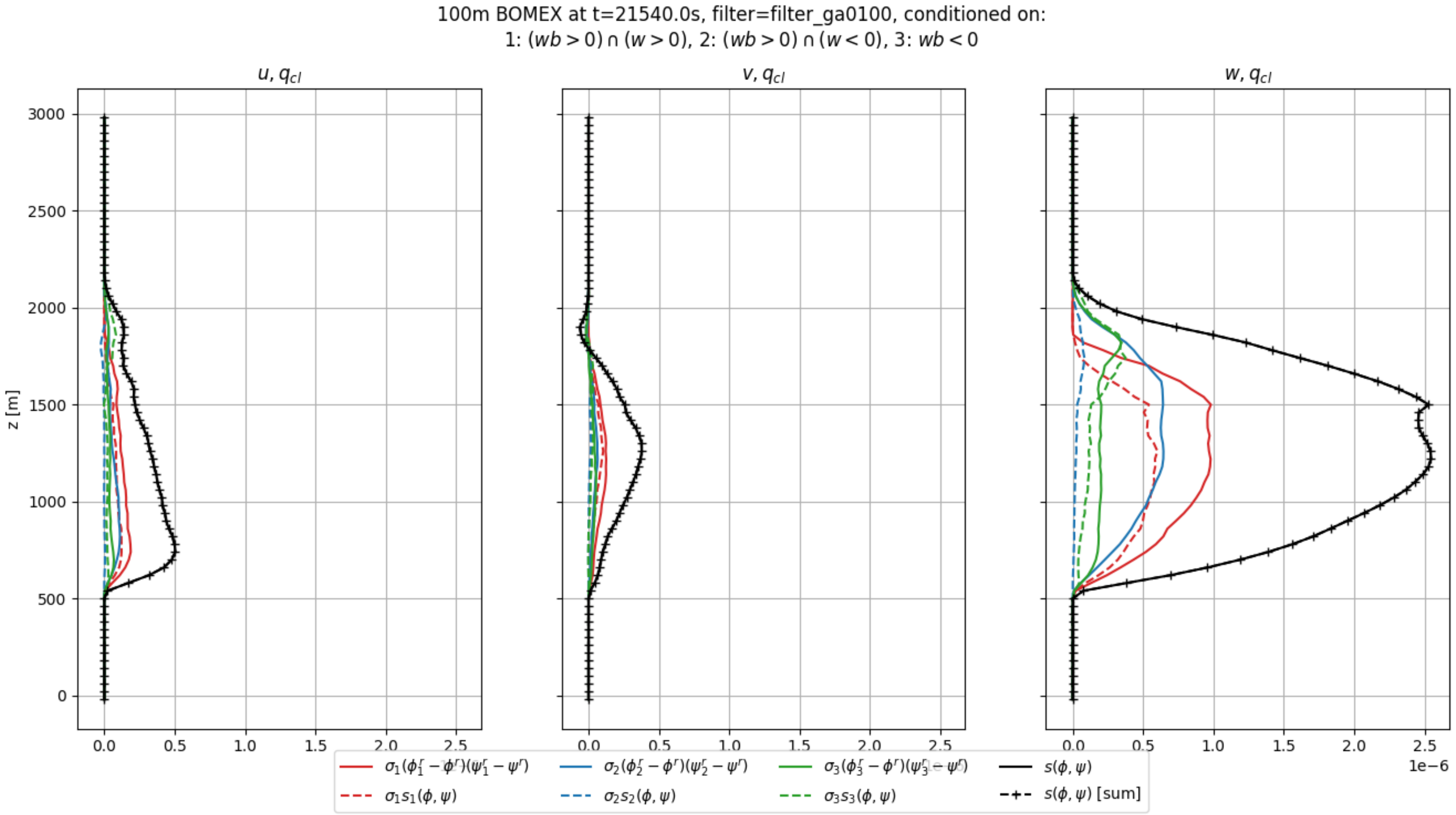
Domain-averaged: $s(u, q_{cl})$ $s(v, q_{cl})$ $s(w, q_{cl})$

Conditional filtering: vertical fluxes vs. scale

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} = 400\ \mathrm{m}\ (= 2\Delta x)$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

— total
— coherent
- - incoherent
-+- total
(Siebesma identity)



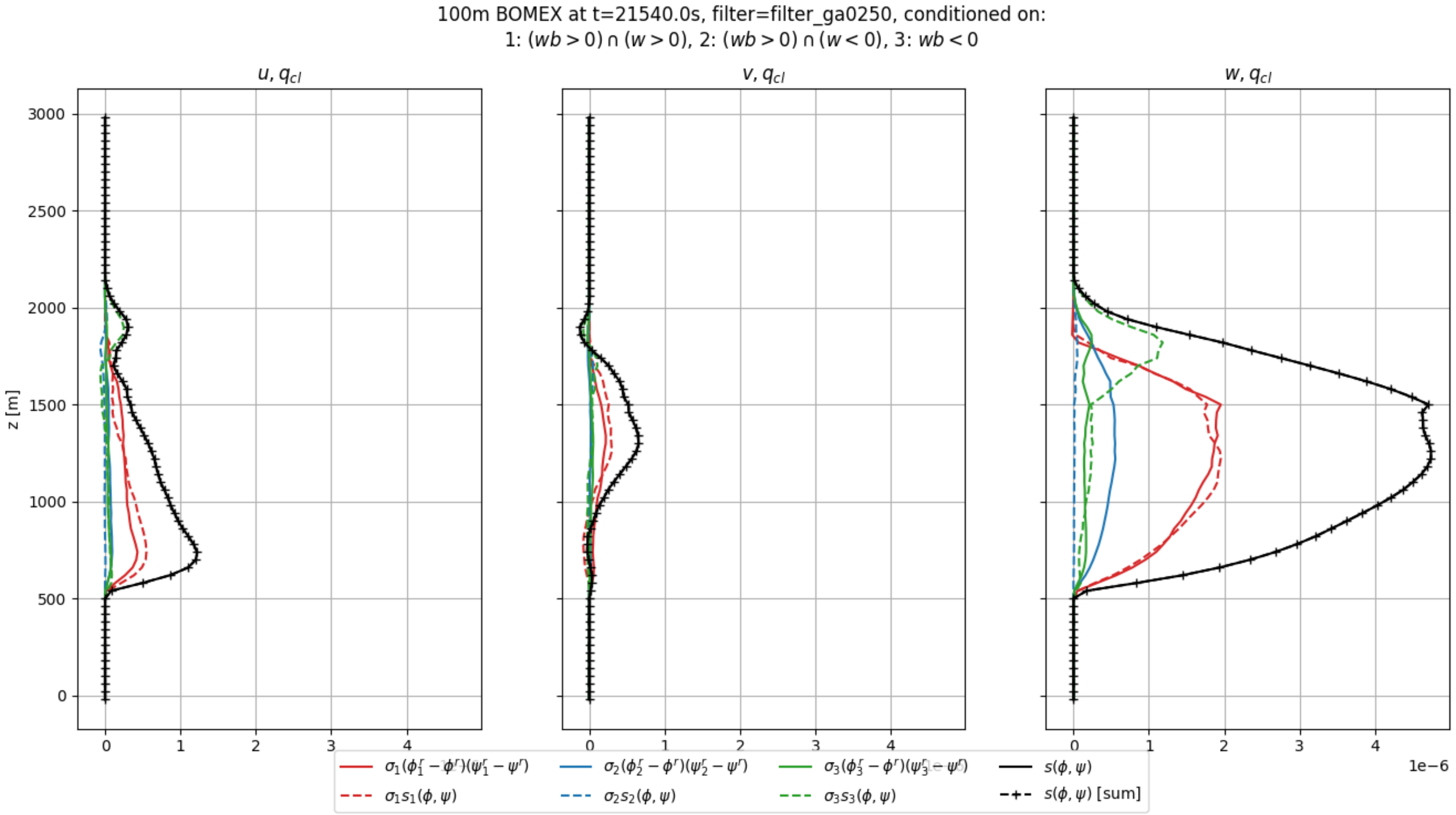
Domain-averaged: $s(u, q_{cl})$ $s(v, q_{cl})$ $s(w, q_{cl})$

Conditional filtering: vertical fluxes vs. scale

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_{\mathrm{f}} = 1000\ \mathrm{m}\ (= 10\Delta x)$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

━ total
━ coherent
- - incoherent
╋╋ total
(Siebesma identity)



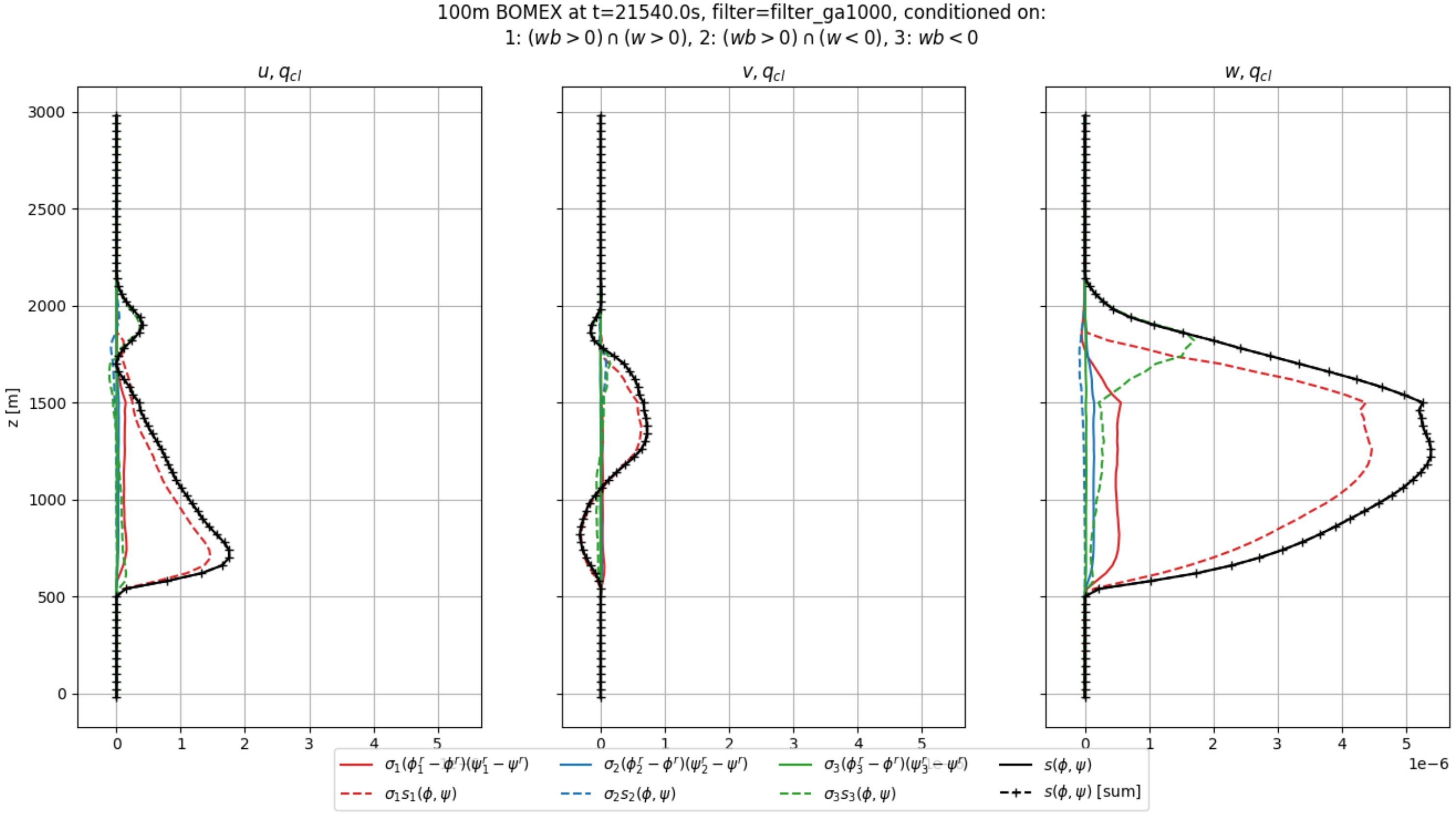
Domain-averaged: $s(u, q_{cl})$ $s(v, q_{cl})$ $s(w, q_{cl})$

Conditional filtering: vertical fluxes vs. scale

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_f = 4000\ \mathrm{m}\ (= 10\Delta x)$

Condition 1:
$wb > 0\ \mathrm{AND}\ w > 0$
Condition 2:
$wb > 0\ \mathrm{AND}\ w < 0$
Condition 3:
$wb < 0$

━━ total
━━ coherent
‒ ‒ incoherent
┿┿ total
(Siebesma identity)



Domain-averaged: $s(u, q_{cl})$ $s(v, q_{cl})$ $s(w, q_{cl})$

Coherent part >= half of total flux until $\ell_f >$ cloud scale. Incoherent part important at all scales!

This split is partition-dependent: e.g. cloudy updraft partition coherent q_cl flux is dominant at all scales (but incoherent part dominates other fluxes e.g. momentum).

Conditional filtering: ratio of horizontal to vertical fluxes vs. scale

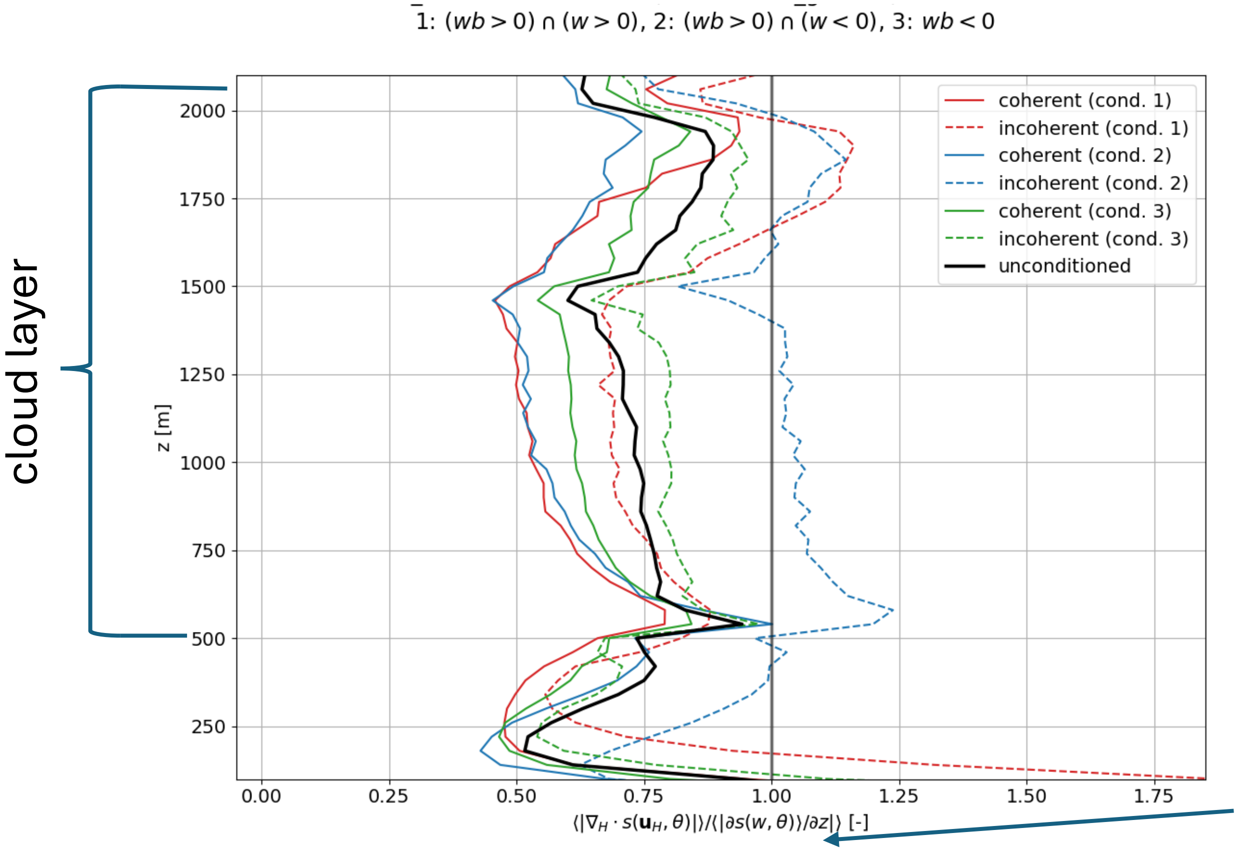
- However it is not the fluxes that directly enter the equations of motion; it is the flux divergences.
- Therefore we investigate the ratio of the magnitude of the horizontal part of the flux divergence to the magnitude of the vertical part of the flux divergence.

Conditional filtering: ratio of horizontal to vertical fluxes vs. scale

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} = 200\ \mathrm{m}\ (= 2\Delta x)$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

▬ total
— coherent
- - incoherent



Note:
Ratio ~ 0 => 1D assumption good (RANS limit)
Ratio = 1 => isotropic eddies (LES limit)
Ratio O(1) but <1 => horizontal fluxes important but eddies anisotropic

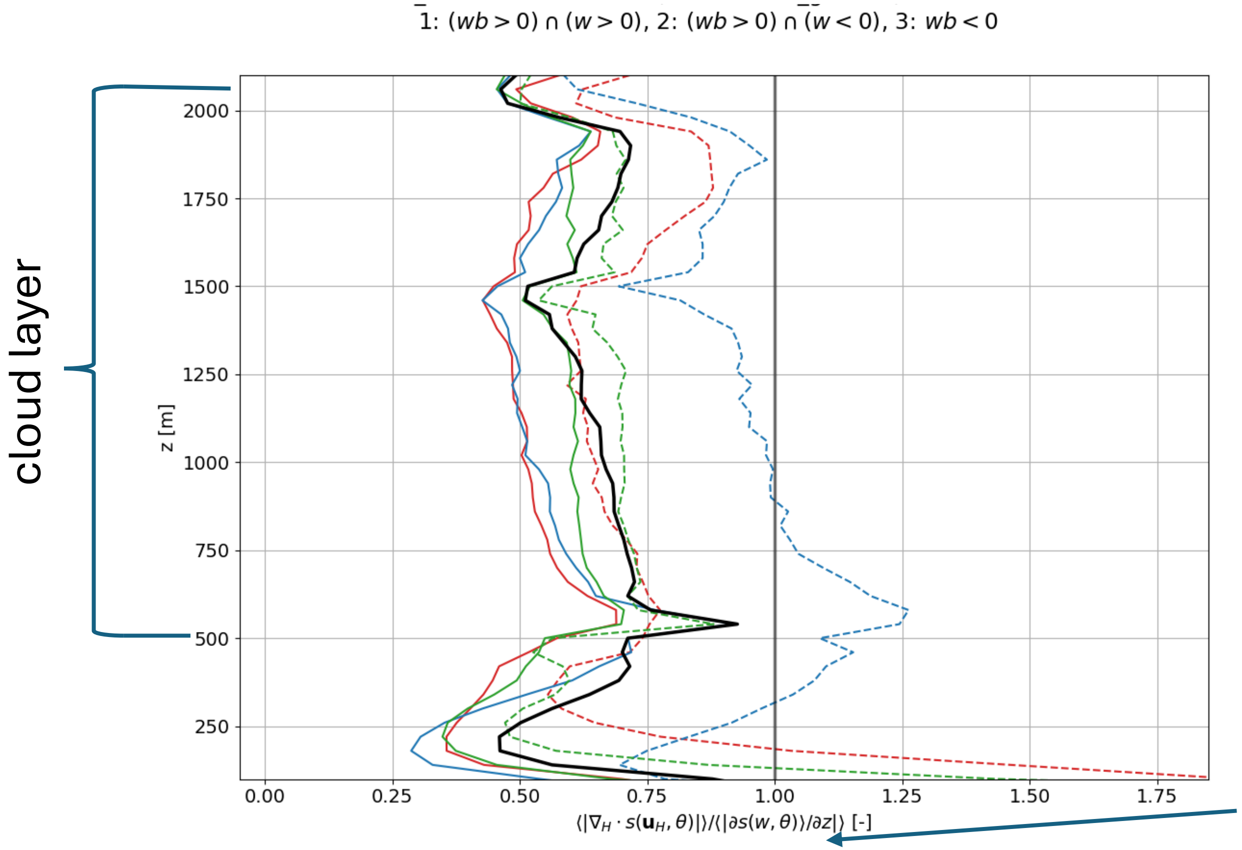
$$\frac{\text{Domain-averaged absolute value of horizontal flux divergence}}{\text{Domain-averaged absolute value of vertical flux divergence}}$$

Conditional filtering: ratio of horizontal to vertical fluxes vs. scale

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} = 400\ \mathrm{m}\ (= 4\Delta x)$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

▬ total
▬ coherent
╌ incoherent



Domain-averaged absolute value of horizontal flux divergence

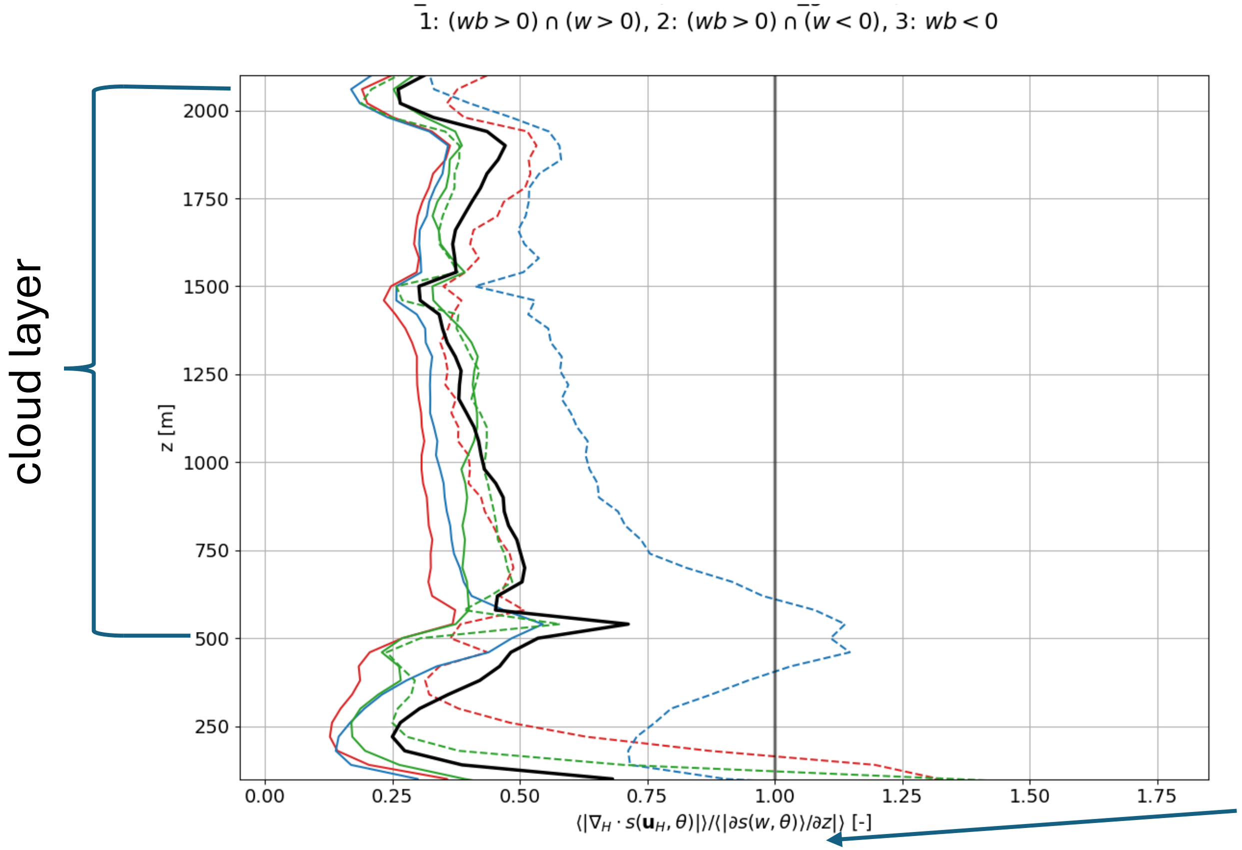
Domain-averaged absolute value of vertical flux divergence

Conditional filtering: ratio of horizontal to vertical fluxes vs. scale

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} = 1000\ \mathrm{m}\ (= 10\Delta x)$

Condition 1:
$wb > 0 \text{ AND } w > 0$
Condition 2:
$wb > 0 \text{ AND } w < 0$
Condition 3:
$wb < 0$

━ total
━ coherent
╍ incoherent



Horizontal fluxes $\gtrsim 0.5$ vert flux. until $\ell_\mathrm{f} \gtrsim \ell_\mathrm{ic}$

$$\frac{\text{Domain-averaged absolute value of horizontal flux divergence}}{\text{Domain-averaged absolute value of vertical flux divergence}}$$

Conditional filtering: ratio of horizontal to vertical fluxes vs. scale

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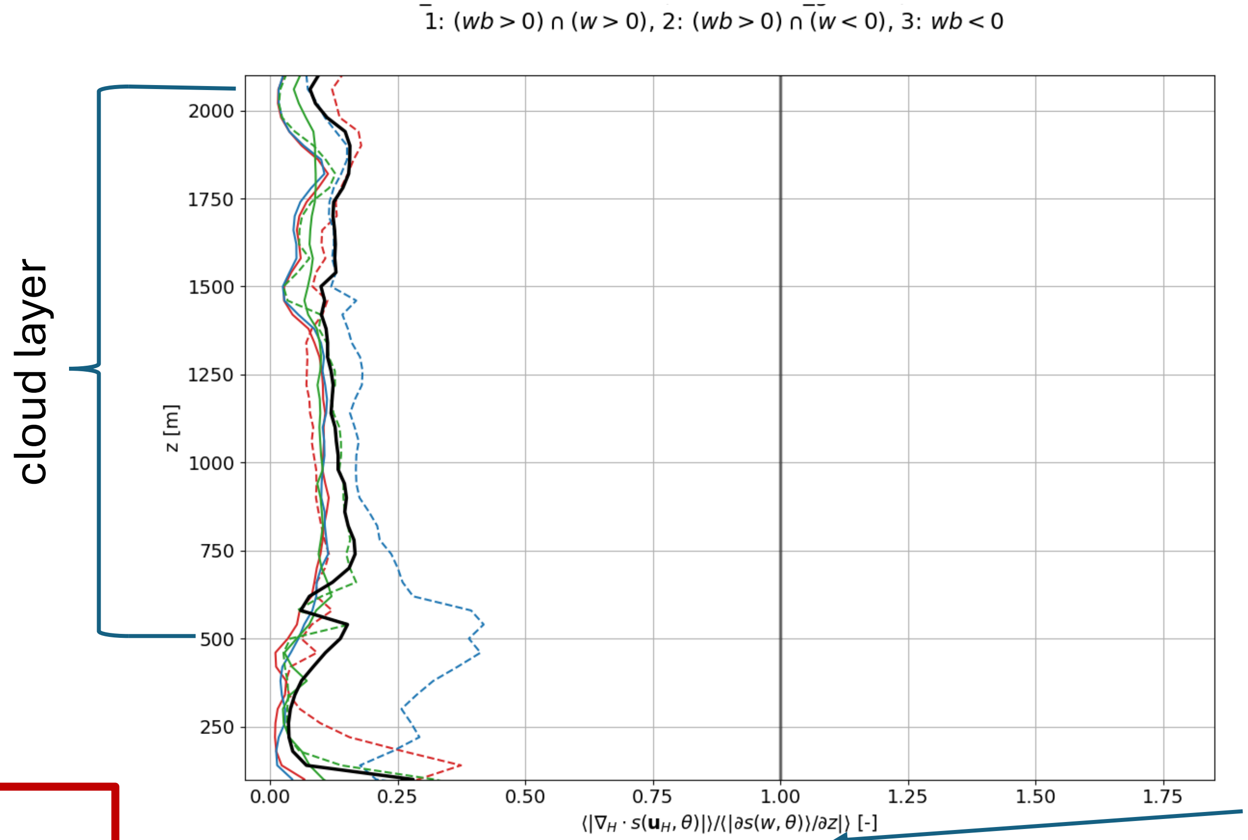
BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_{\mathrm{f}} = 4000\ \mathrm{m}\ (= 40\Delta x)$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

▬ total
▬ coherent
╌ incoherent



Horizontal fluxes same order of magnitude as vertical fluxes (i.e. hor. flux $\gtrsim$ 0.1 vert. flux) until $\ell_{\mathrm{f}} \gtrsim 4 - 5\ell_{\mathrm{ic}}$

$$\frac{\text{Domain-averaged absolute value of horizontal flux divergence}}{\text{Domain-averaged absolute value of vertical flux divergence}}$$

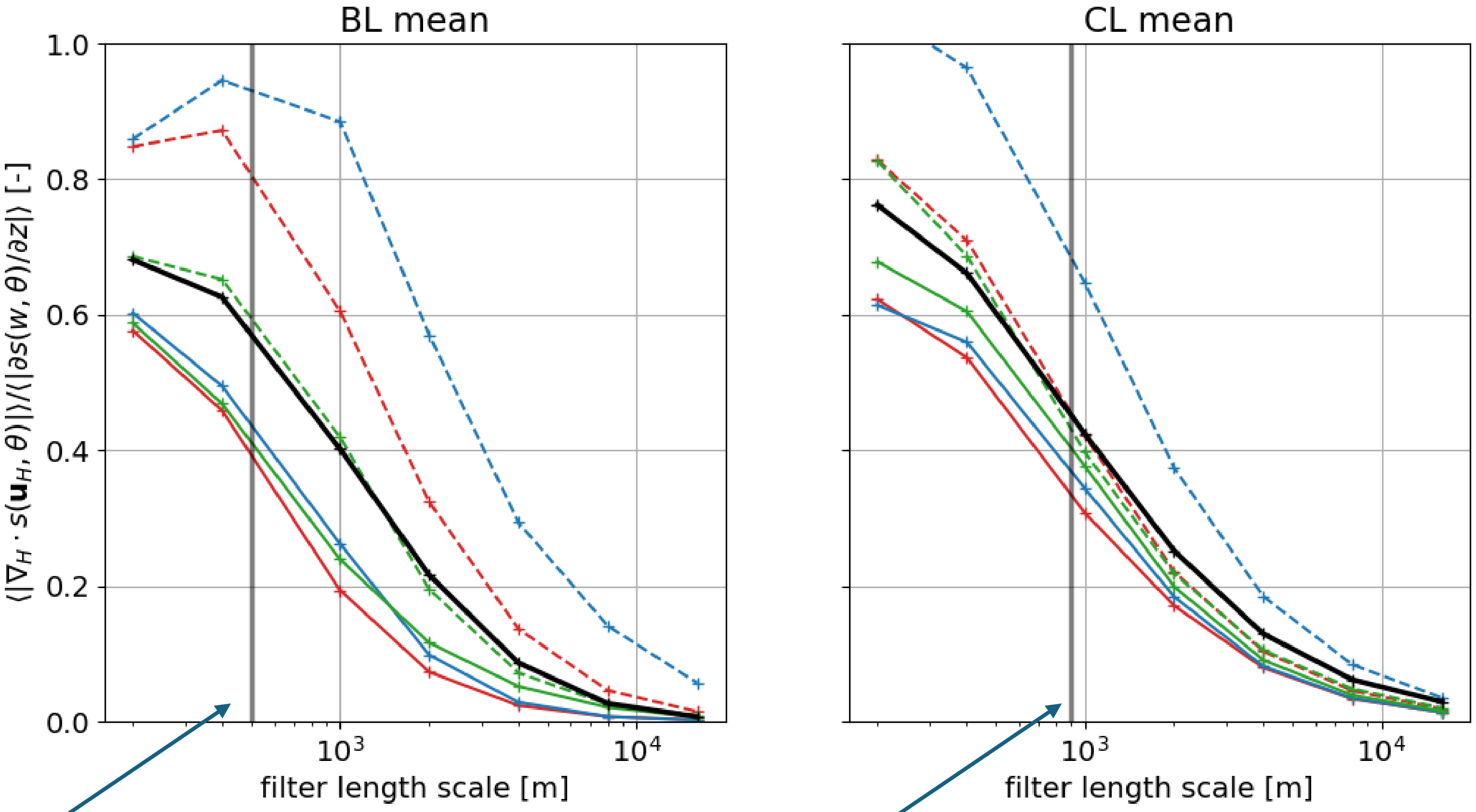
Note: similar behaviour regardless of partition choice or flux choice

Conditional filtering: horizontal fluxes vs. scale

BOMEX, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} \in [200{,}16000]$ m

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

— total
— coherent
– – incoherent



BL depth

Cloud spacing

Horizontal fluxes $\gtrsim$ 0.5 vertical fluxes until $\ell_\mathrm{f} \gtrsim \ell_\mathrm{ic}$

Ratio consistently lower for coherent part of fluxes & higher for incoherent part relative to full flux divergence

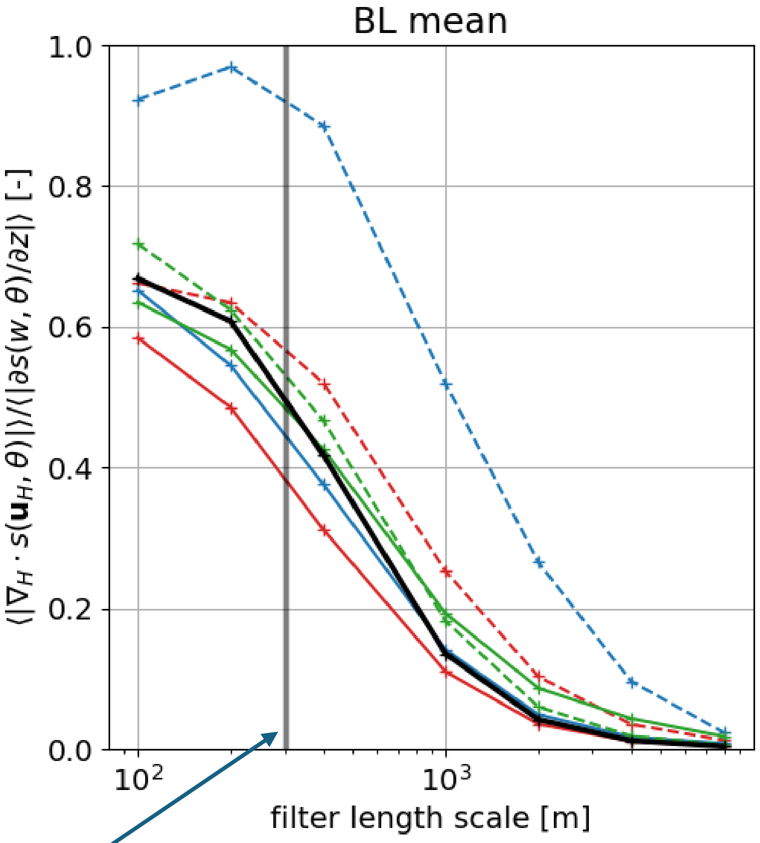
Conditional filtering: horizontal fluxes vs. scale

ARM, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} \in [100{,}8000]\ \mathrm{m}$

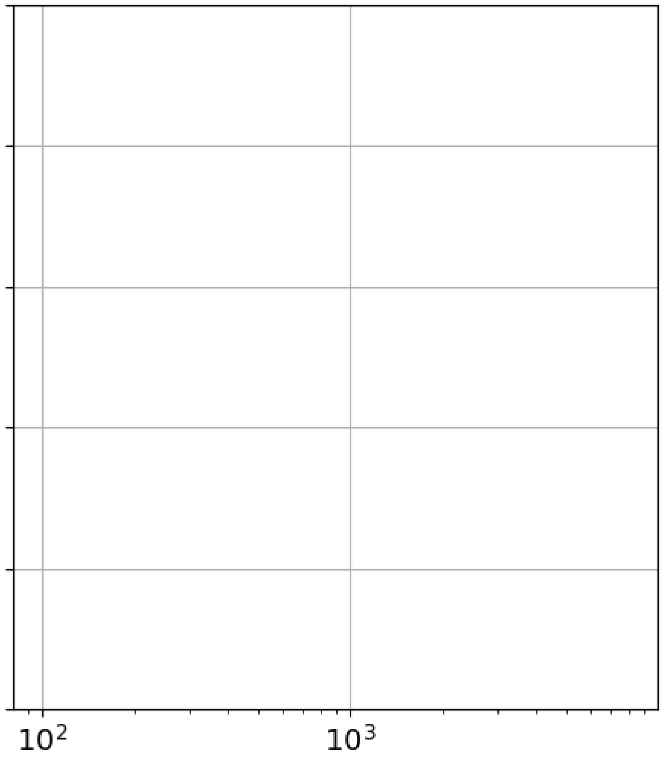
Condition 1:
$wb > 0 \text{ AND } w > 0$
Condition 2:
$wb > 0 \text{ AND } w < 0$
Condition 3:
$wb < 0$

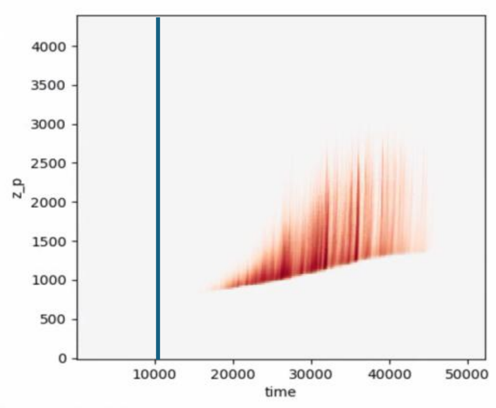
━ total
━ coherent
╌ incoherent

Hour 3



BL depth





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ParaChute Programme

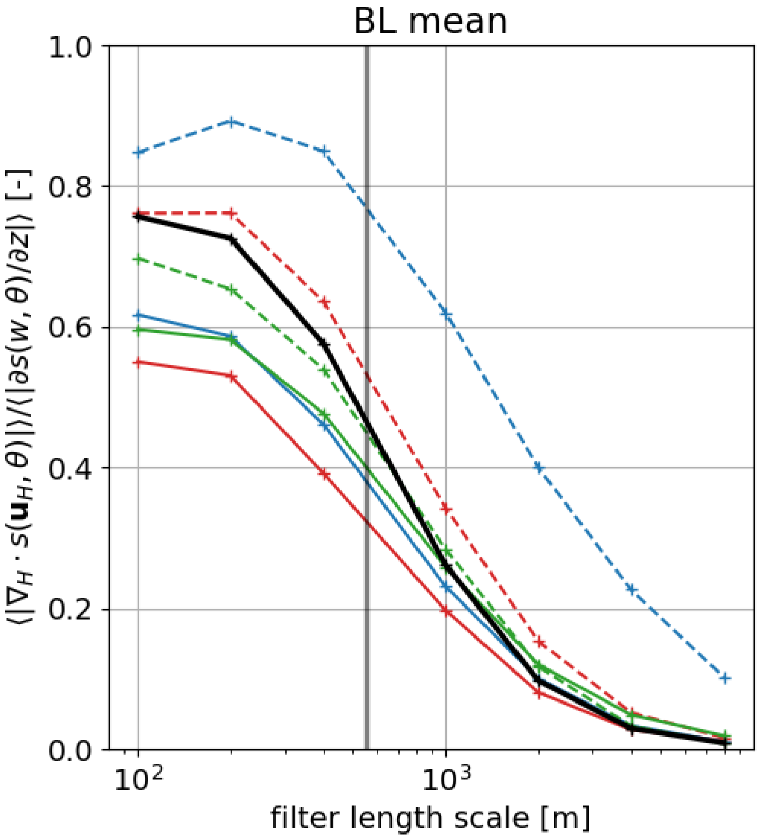
Conditional filtering: horizontal fluxes vs. scale

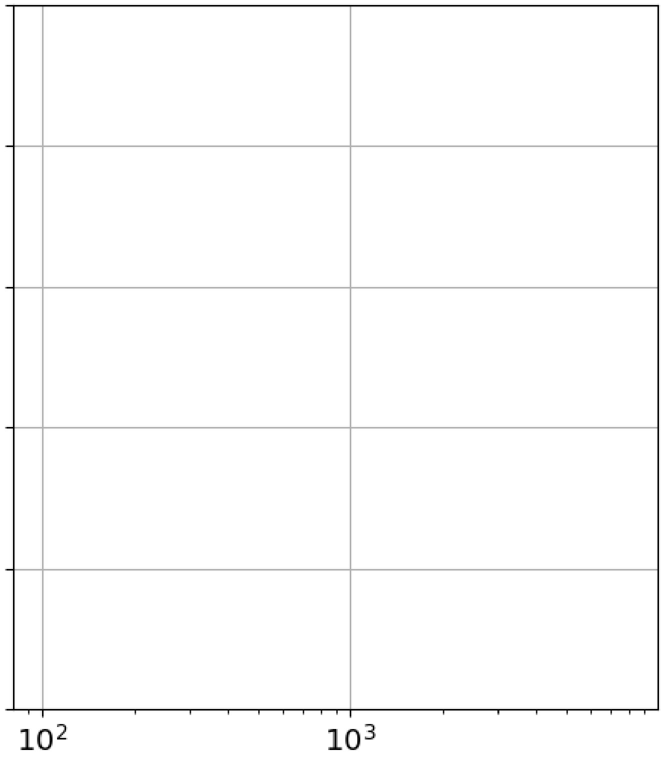
ARM, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} \in [100{,}8000]$ m

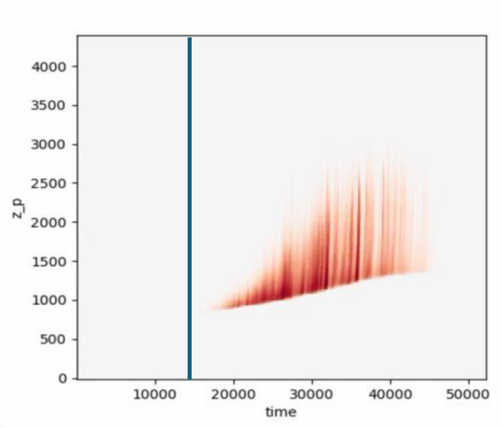
Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

━ total
━ coherent
╍ incoherent

Hour 4







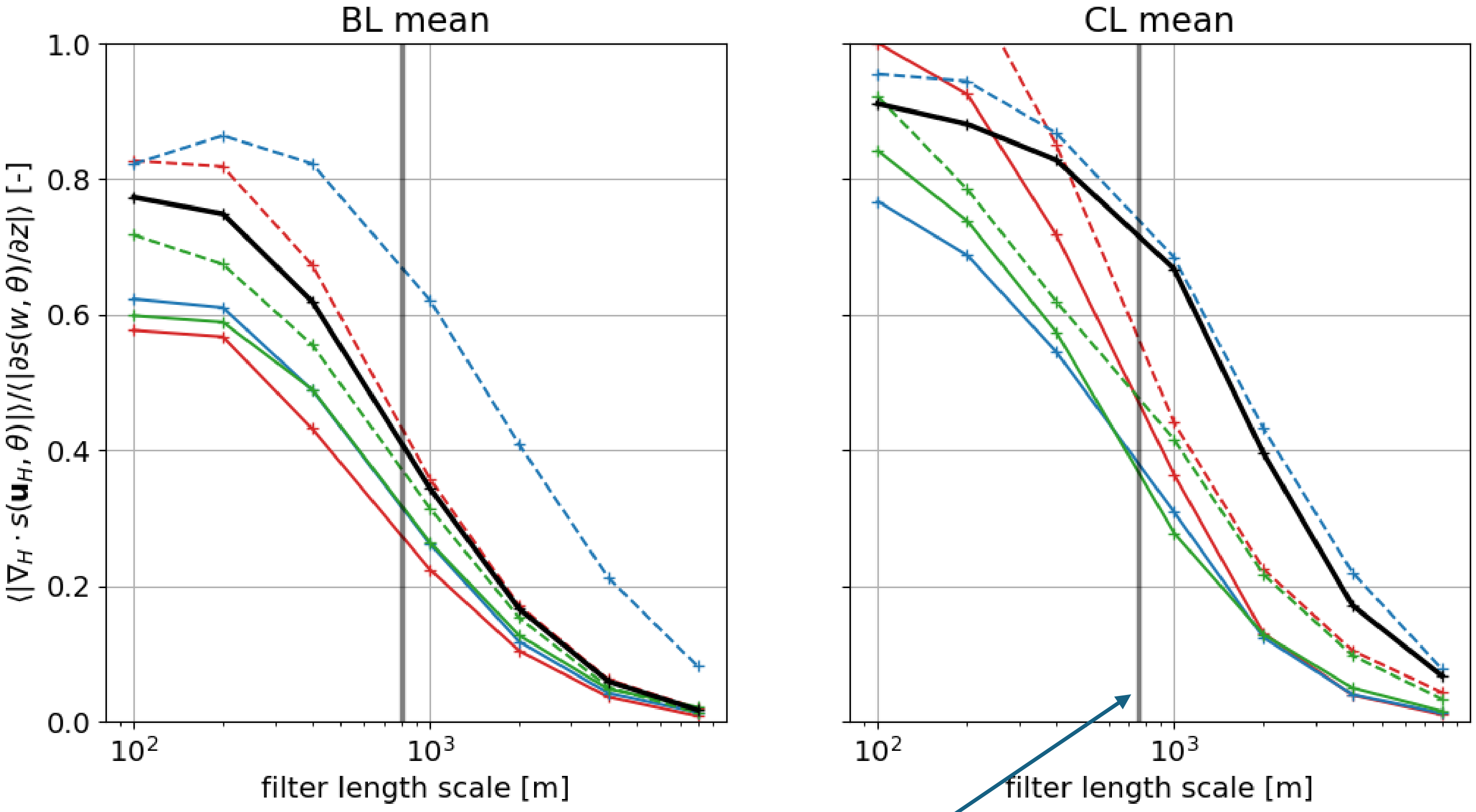
Conditional filtering: horizontal fluxes vs. scale

ARM, conditioned on buoyancy flux, Gaussian filter $\ell_{\mathrm{f}} \in [100,8000]\ \mathrm{m}$

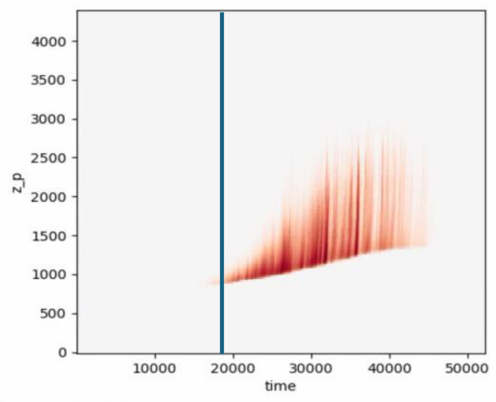
Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

▬ total
▬ coherent
╍ incoherent

Hour 5



Cloud spacing



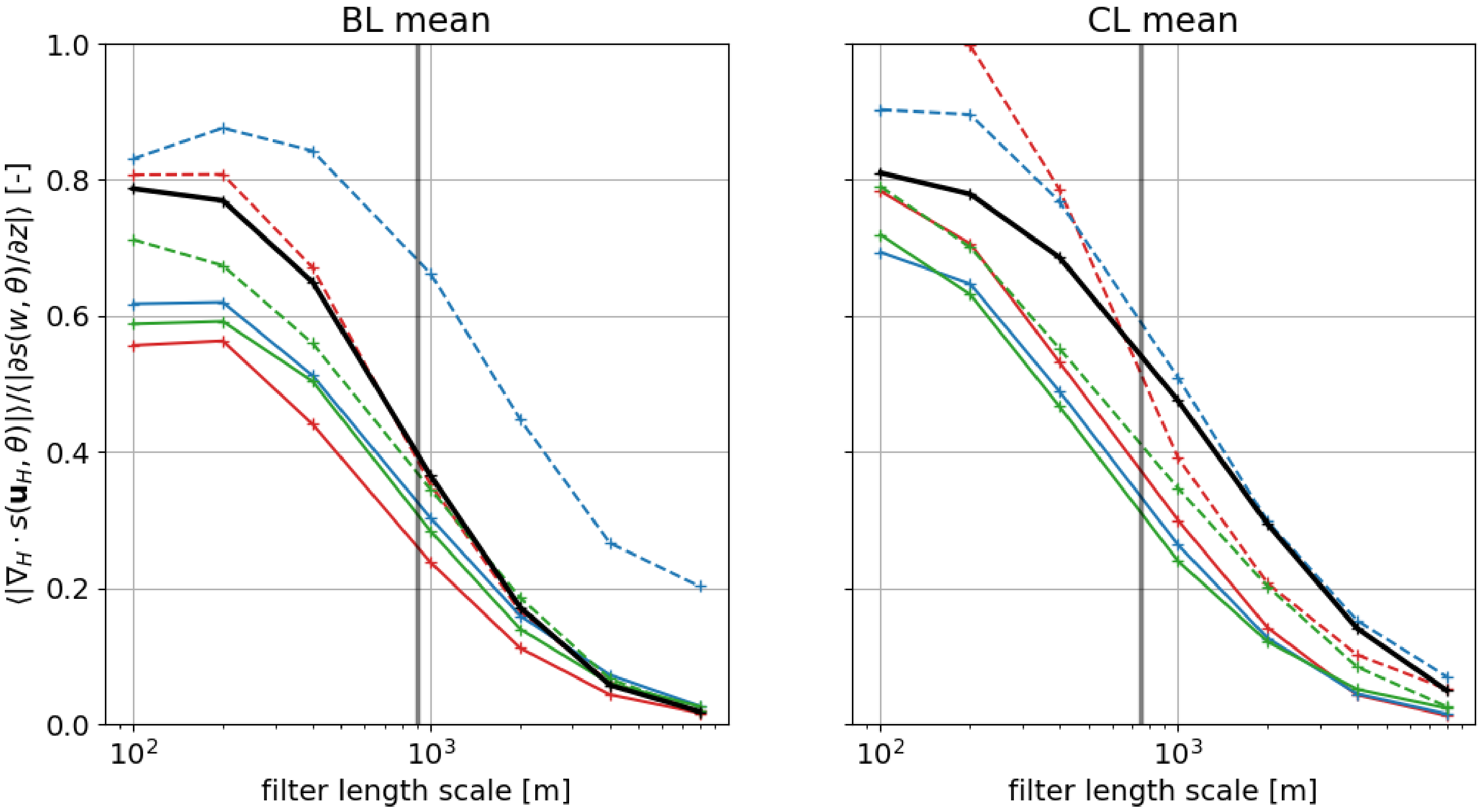
Conditional filtering: horizontal fluxes vs. scale

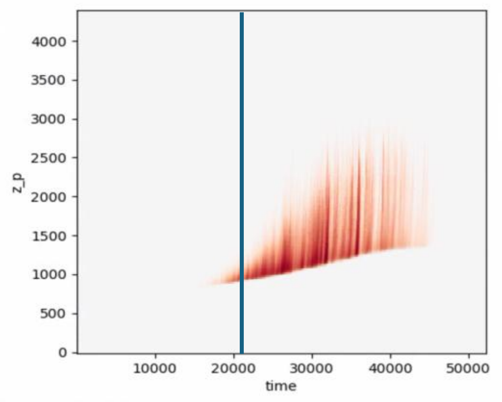
ARM, conditioned on buoyancy flux, Gaussian filter $\ell_{\mathrm{f}} \in [100{,}8000]\ \mathrm{m}$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

— total
— coherent
- - incoherent

Hour 6





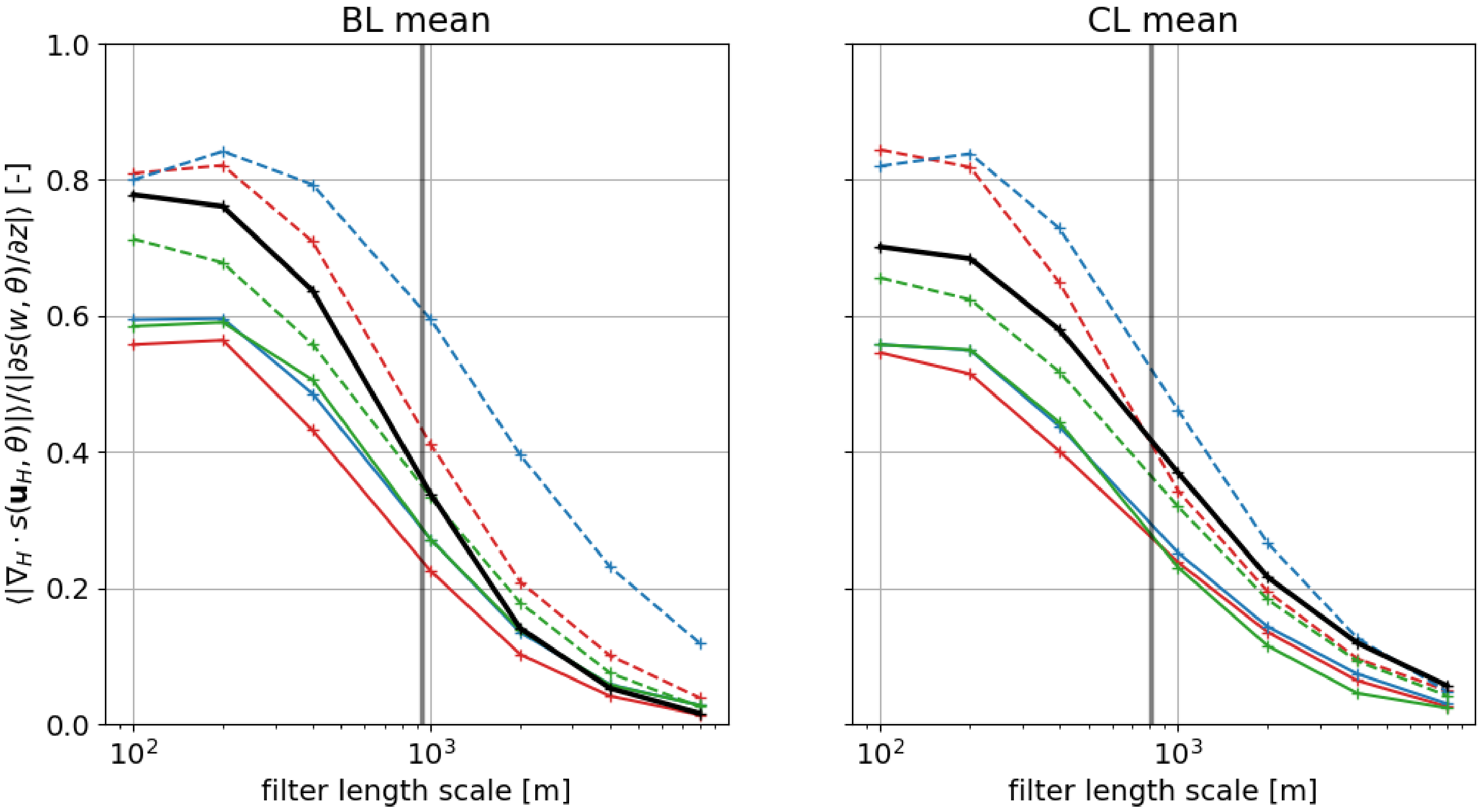
Conditional filtering: horizontal fluxes vs. scale

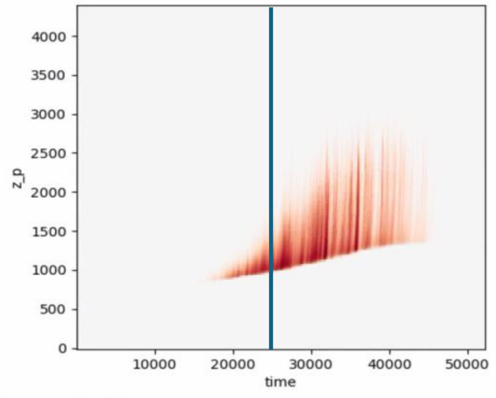
ARM, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} \in [100{,}8000]$ m

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

— total
— coherent
- - incoherent

Hour 7





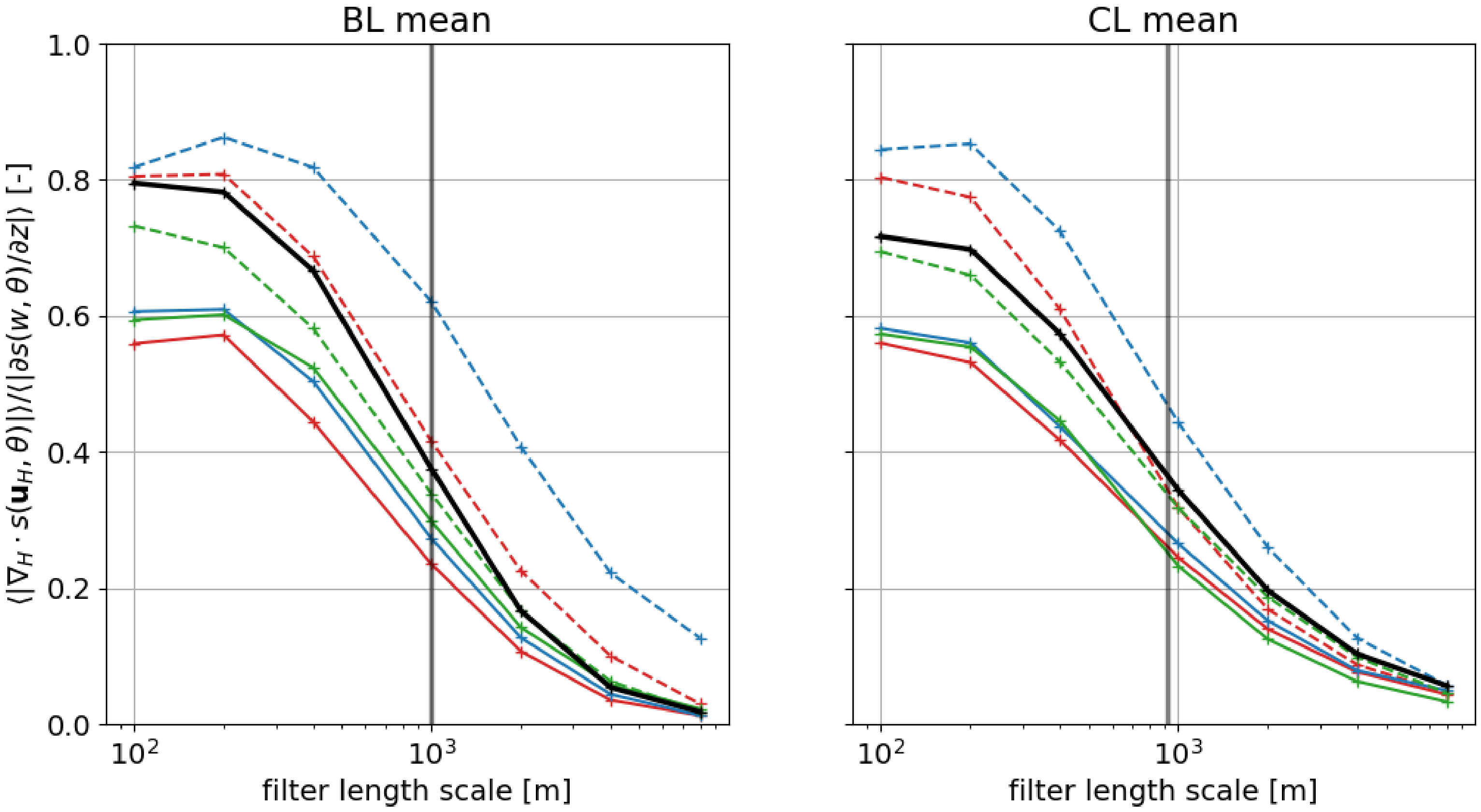
Conditional filtering: horizontal fluxes vs. scale

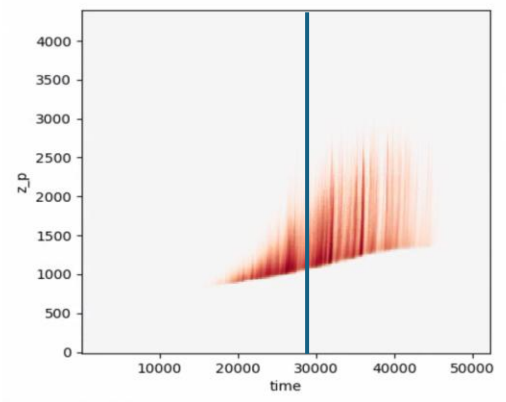
ARM, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} \in [100{,}8000]\ \mathrm{m}$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

━ total
━ coherent
╌ incoherent

Hour 8





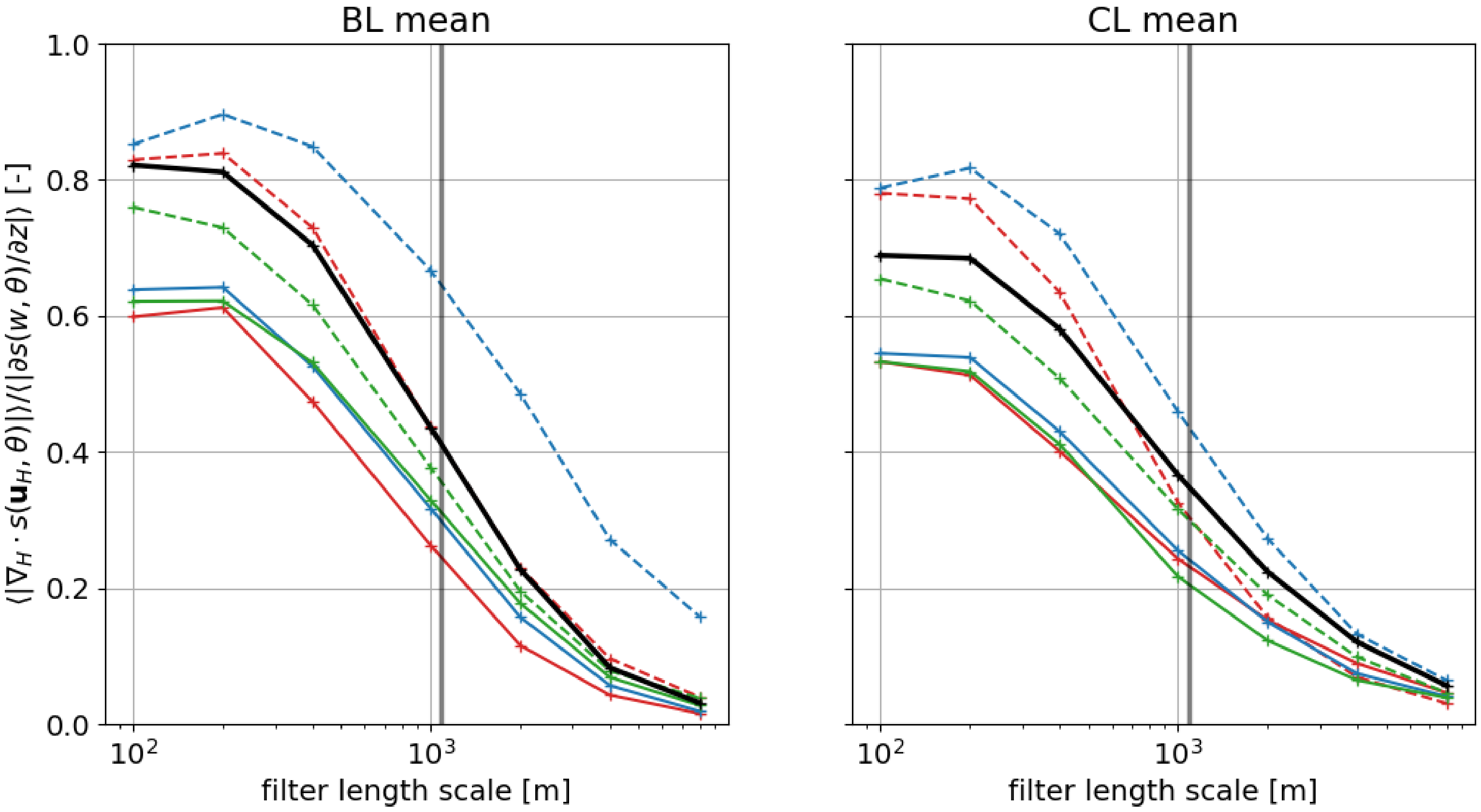
Conditional filtering: horizontal fluxes vs. scale

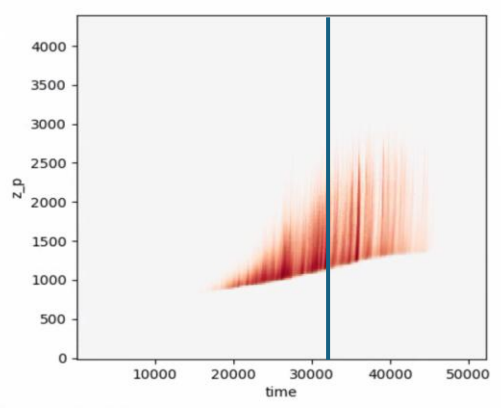
ARM, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} \in [100{,}8000]\ \mathrm{m}$

Condition 1:
$wb > 0\ \mathrm{AND}\ w > 0$
Condition 2:
$wb > 0\ \mathrm{AND}\ w < 0$
Condition 3:
$wb < 0$

total
coherent
incoherent

Hour 9





Conditional filtering: horizontal fluxes vs. scale

ARM, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} \in [100{,}8000]$ m

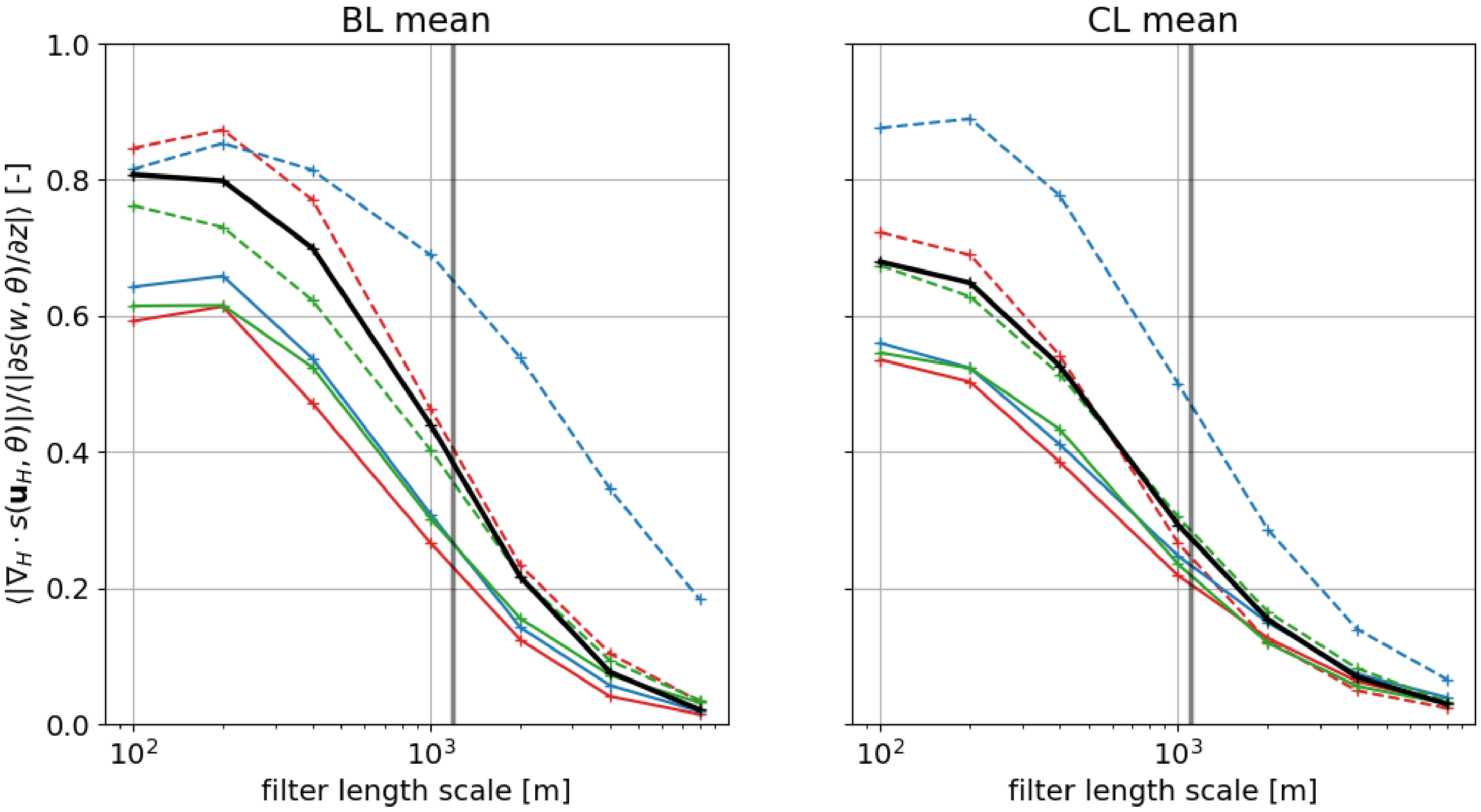
Condition 1:
$wb > 0$ AND $w > 0$

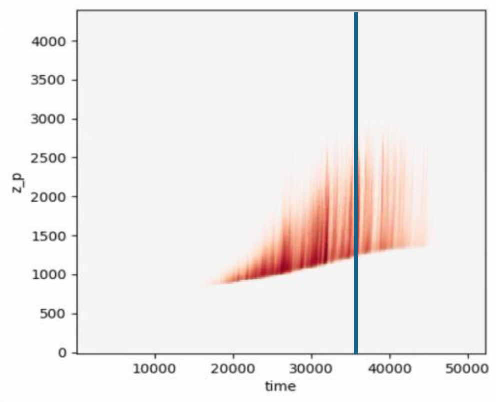
Condition 2:
$wb > 0$ AND $w < 0$

Condition 3:
$wb < 0$

━━ total
━━ coherent
‐ ‐ ‐ incoherent

Hour 10





Conditional filtering: horizontal fluxes vs. scale

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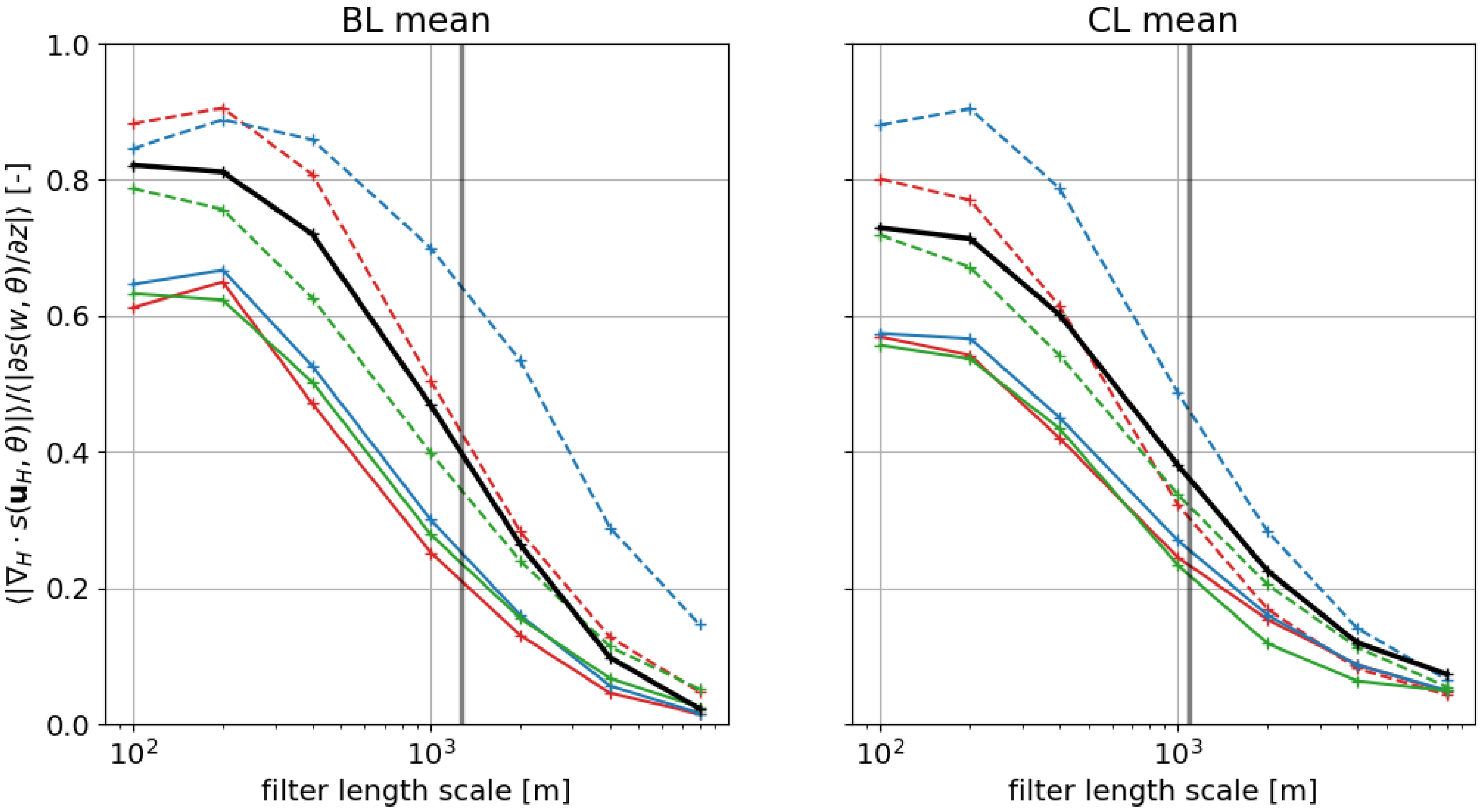
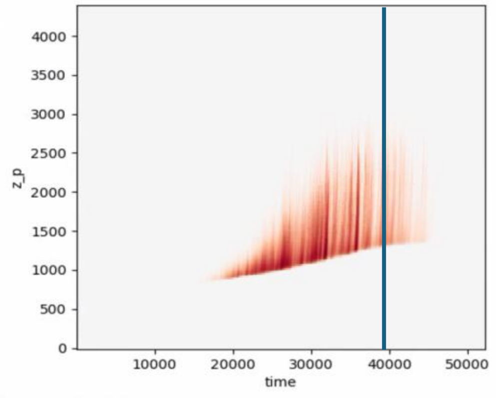
ARM, conditioned on buoyancy flux, Gaussian filter $\ell_{\mathrm{f}} \in [100{,}8000]\ \mathrm{m}$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

- total
- coherent
- incoherent

Hour 11





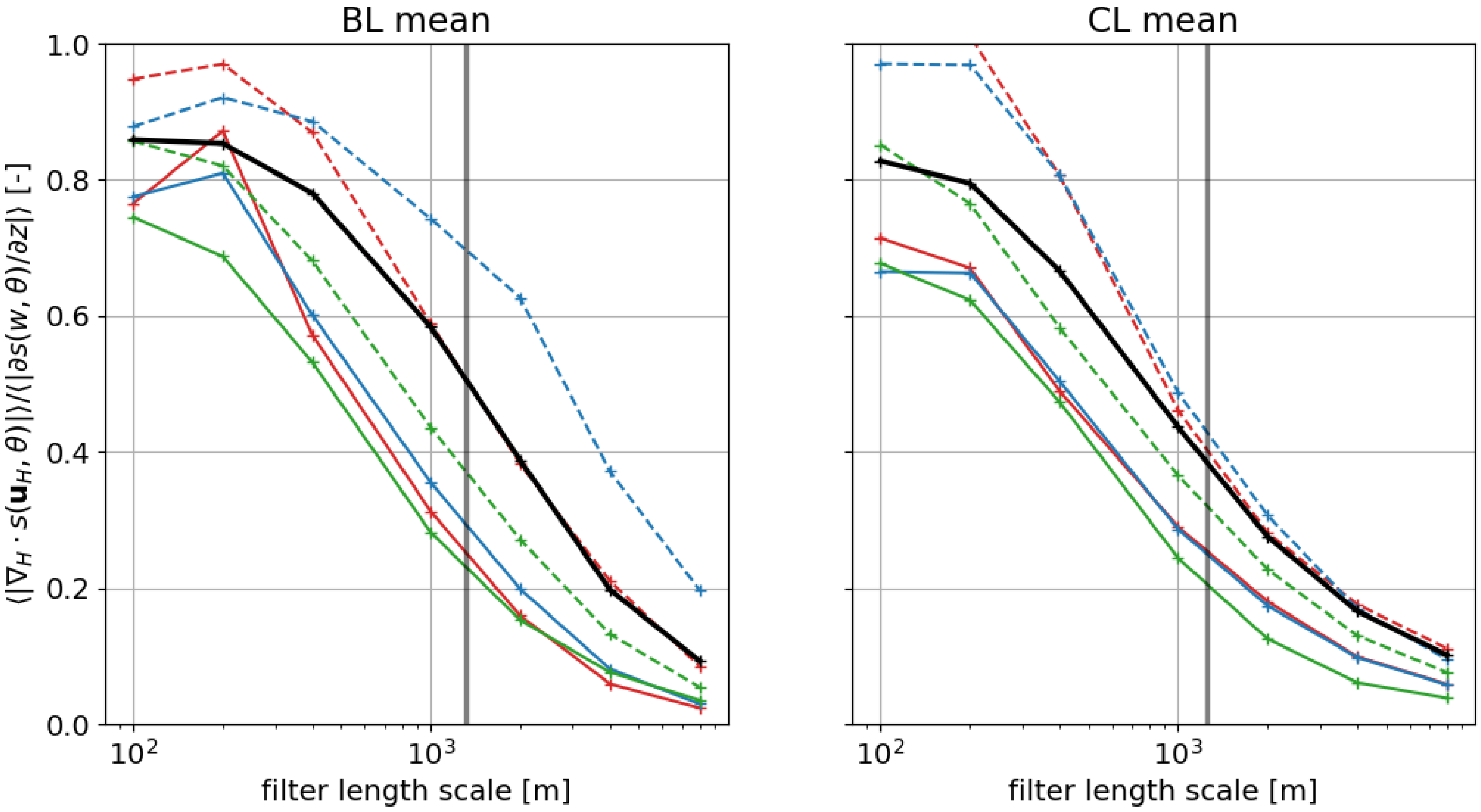
Conditional filtering: horizontal fluxes vs. scale

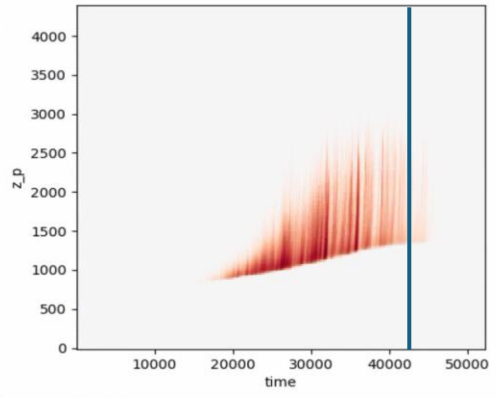
ARM, conditioned on buoyancy flux, Gaussian filter $\ell_{\mathrm{f}} \in [100{,}8000]\ \mathrm{m}$

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

▬ total
▬ coherent
╌ incoherent

Hour 12





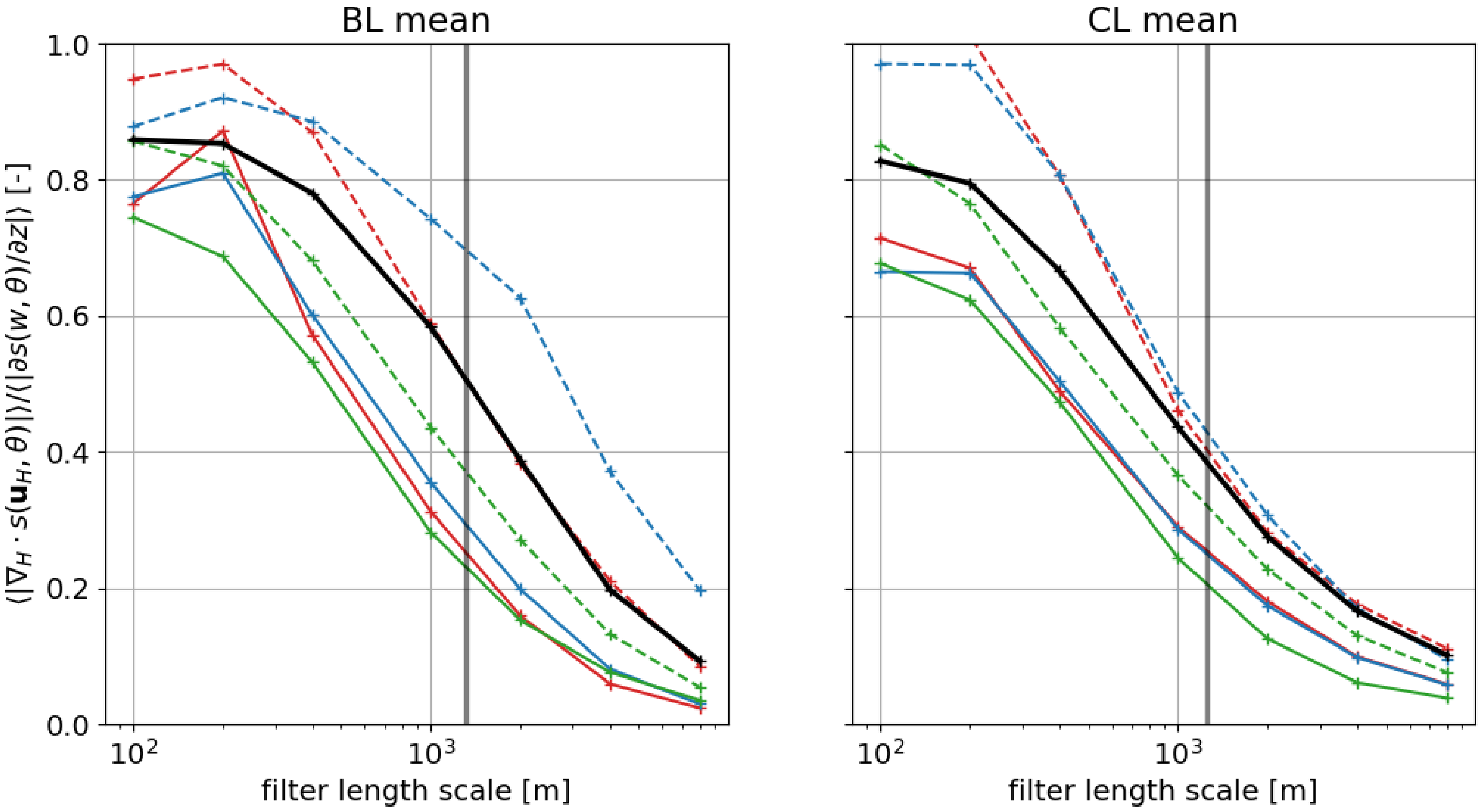
Conditional filtering: horizontal fluxes vs. scale

ARM, conditioned on buoyancy flux, Gaussian filter $\ell_\mathrm{f} \in [100,8000]$ m

Condition 1:
$wb > 0$ AND $w > 0$
Condition 2:
$wb > 0$ AND $w < 0$
Condition 3:
$wb < 0$

━ total
━ coherent
- - incoherent

Hour 12



Similar results to BOMEX across time for ARM.

Again, horizontal fluxes $\gtrsim 0.5$ vertical fluxes until $\ell_\mathrm{f} \gtrsim \ell_\mathrm{ic}$ (cloud layer).

Ratio consistently lower for coherent part of flux.

Summary

- Boundary layer and convection modellers approach the grey zone from different directions; we wish to transition between these different representations in a smooth and physically consistent way (could view as an asymptotic matching problem).
- Conditional spatial filtering provides a rigorous framework for calculating the quantities that appear in mass flux-type models in such a way that:
    - they are directly comparable with quantities appearing in the unconditionally-filtered higher-order equations;
    - they obey desirable mathematical properties, providing "sanity checks".
- Preliminary work shows the usefulness of this approach for calculating various quantities of interest, e.g. the dependence of horizontal vs. vertical fluxes with filtering scale.
    - Ratio of magnitude of horizontal to vertical fluxes follows sigmoid curve with resolution.
    - Horizontal fluxes become similarly important to vertical fluxes at around the inter-cloud spacing.
    - The ratio of horizontal-to-vertical fluxes is slightly smaller for coherent parts of fluxes than total flux in both BL and cloud layer, i.e. the "mass flux" split slightly delays the onset of 3D in the coherent part of the flux, but hastens the onset for the incoherent part.
- Conclusion: some representation of horizontal fluxes will be vital for the convective grey zone regardless of parametrization approach!
    - …but within a mass flux-type scheme we might be able to get away with confining this to residual subgrid fluxes rather than a 3D mass flux.

Additional slides

Relating filtering ("turbulence" approach) to conditional filtering ("mass flux" approach)

- As both frameworks are equivalent, but currently applied to different regimes in CoMorph and 3DTE, it makes sense to attempt to find common ground between the approaches by writing them both within the same framework.
- As an example, let's look at the vertical velocity variance equation. In the higher-moment closure approach this looks like:

$$\frac{\mathrm{D}s(w,w)}{\mathrm{D}t} = -\boldsymbol{\nabla}\cdot s(w,w,\mathbf{u}) - \frac{2}{\rho}\frac{\partial}{\partial z}s(w,p) + \frac{2}{\rho}s\left(p,\frac{\partial w}{\partial z}\right) - 2s(w,\mathbf{u})\cdot\boldsymbol{\nabla}w^{\mathrm{r}} + 2s(w,b). \qquad (9)$$

1. Material derivative following resolved velocity

2. Transport of variance due to velocity skewness

3. Transport of variance due to pressure-velocity correlations

4. "Pressure scrambling"

5. Creation/destruction of variance by shear (also = inter-scale transfer of vertical part of TKE)

6. Creation/destruction of variance by subgrid buoyancy fluctuations

Relating filtering ("turbulence" approach) to conditional filtering ("mass flux" approach)

- In the higher-moment closure approach the vertical velocity variance equation is:

$$\frac{\mathrm{D}s(w,w)}{\mathrm{D}t} = -\boldsymbol{\nabla}\cdot s(w,w,\mathbf{u}) - \frac{2}{\rho}\frac{\partial}{\partial z}s(w,p) + \frac{2}{\rho}s\left(p,\frac{\partial w}{\partial z}\right) - 2s(w,\mathbf{u})\cdot\boldsymbol{\nabla}w^{\mathrm{r}} + 2s(w,b). \tag{9}$$

- However, using eq. (8) we can also write:

$$\frac{\mathrm{D}s(w,w)}{\mathrm{D}t} = \sum_i \frac{\mathrm{D}}{\mathrm{D}t}\left[\sigma_i s_i(w,w)\right] + 2\sum_i (w_i^{\mathrm{r}} - w^{\mathrm{r}})\frac{\mathrm{D}}{\mathrm{D}t}\left[\sigma_i(w_i^{\mathrm{r}} - w^{\mathrm{r}})\right] - \sum_i (w_i^{\mathrm{r}} - w^{\mathrm{r}})(w_i^{\mathrm{r}} - w^{\mathrm{r}})\frac{\mathrm{D}\sigma_i}{\mathrm{D}t} \tag{10}$$

Relating filtering (“turbulence” approach) to conditional filtering (“mass flux” approach)

- After some algebra we arrive at the exact result:

$$
\begin{aligned}
\rho_{\mathrm{R}}\frac{\mathrm{D}s(w,w)}{\mathrm{D}t} = \sum_i \Bigg\{ \\
& - \boldsymbol{\nabla}\cdot\left(\mathrm{M}_i(w_i')^2 + 2\rho_{\mathrm{R}}\sigma_i w_i' s_i(\mathbf{u},w) + \mathrm{M}_i s_i(w,w) + \rho_{\mathrm{R}}\sigma_i s_i(w,w,\mathbf{u})\right) \\
& - 2\rho_{\mathrm{R}}\sigma_i s_i(\mathbf{u},w)\cdot\boldsymbol{\nabla}w^{\mathrm{r}} - 2w_i'\mathrm{M}_i\cdot\boldsymbol{\nabla}w^{\mathrm{r}} + \cancel{0} + \cancel{2\rho_{\mathrm{R}}w_i'\sigma_i\boldsymbol{\nabla}\cdot s(\mathbf{u},w)} \\
& + 2\rho_{\mathrm{R}}\sigma_i s_i(w,b) + 2\rho_{\mathrm{R}}\sigma_i w_i' b_i' \\
& - 2\frac{\partial}{\partial z}\sigma_i\left[s_i(w,p) + w_i'p_i'\right] + 2\sigma_i\left[s_i\left(\frac{\partial w}{\partial z},p\right) + p_i'\left(\frac{\partial w}{\partial z}\right)_i'\right] \\
& \cancel{+ 2\left(wp\frac{\partial I_i}{\partial z}\right)^{\mathrm{r}} - 2w^{\mathrm{r}}p^{\mathrm{r}}\frac{\partial\sigma_i}{\partial z} - 2w^{\mathrm{r}}s\left(p,\frac{\partial I_i}{\partial z}\right) - 2p^{\mathrm{r}}s\left(w,\frac{\partial I_i}{\partial z}\right)} + \cancel{0} \\
& \cancel{+ \rho_{\mathrm{R}}\left(w^2\left[\mathcal{S}_i^+ - \mathcal{S}_i^-\right]\right)^{\mathrm{r}} - 2\rho_{\mathrm{R}}w^{\mathrm{r}}\left(w\left[\mathcal{S}_i^+ - \mathcal{S}_i^-\right]\right)^{\mathrm{r}}} + \cancel{0} \\
\Bigg\}. \qquad (11)
\end{aligned}
$$

Origins of terms:

blue = $(w_i^{\mathrm{r}} - w^{\mathrm{r}})\frac{\mathrm{D}}{\mathrm{D}t}\left[\sigma_i(w_i^{\mathrm{r}} - w^{\mathrm{r}})\right]$

red = $\frac{\mathrm{D}}{\mathrm{D}t}\left[\sigma_i s_i(w,w)\right]$

green = $(w_i^{\mathrm{r}} - w^{\mathrm{r}})(w_i^{\mathrm{r}} - w^{\mathrm{r}})\frac{\mathrm{D}\sigma_i}{\mathrm{D}t}$

teal = blue + green

violet = blue + red

magenta = blue + green + red

Cancelling terms that sum to zero over all partitions

Relating filtering ("turbulence" approach) to conditional filtering ("mass flux" approach)

- After some algebra we arrive at the exact result:

$$\rho_{\mathrm{R}}\frac{\mathrm{D}s(w,w)}{\mathrm{D}t} = \sum_i \Bigg\{$$

2. $$-\nabla\cdot\left(\color{teal}{\mathbf{M}_i(w_i')^2} \color{black}{+} \color{blue}{2\rho_{\mathrm{R}}\sigma_i w_i' s_i(\mathbf{u},w)} \color{black}{+} \color{red}{\mathbf{M}_i s_i(w,w) + \rho_{\mathrm{R}}\sigma_i s_i(w,w,\mathbf{u})}\right)$$

5. $$\color{purple}{-2\rho_{\mathrm{R}}\sigma_i s_i(\mathbf{u},w)\cdot\nabla w^{\mathrm{r}}} \color{blue}{-2w_i'\mathbf{M}_i\cdot\nabla w^{\mathrm{r}}}$$

6. $$\color{red}{+2\rho_{\mathrm{R}}\sigma_i s_i(w,b)} \color{blue}{+2\rho_{\mathrm{R}}\sigma_i w_i' b_i'}$$

$$-2\frac{\partial}{\partial z}\sigma_i\left[\color{red}{s_i(w,p)} \color{blue}{+ w_i' p_i'}\color{black}{\Big]} + 2\sigma_i\left[\color{red}{s_i\left(\frac{\partial w}{\partial z},p\right)} \color{blue}{+ p_i'\left(\frac{\partial w}{\partial z}\right)_i'}\color{black}{\right]}$$

3.

4.

$$\Bigg\}. \qquad (12)$$

Origins of terms:

blue = $(w_i^{\mathrm{r}} - w^{\mathrm{r}})\frac{\mathrm{D}}{\mathrm{D}t}\left[\sigma_i(w_i^{\mathrm{r}} - w^{\mathrm{r}})\right]$

red = $\frac{\mathrm{D}}{\mathrm{D}t}\left[\sigma_i s_i(w,w)\right]$

green = $(w_i^{\mathrm{r}} - w^{\mathrm{r}})(w_i^{\mathrm{r}} - w^{\mathrm{r}})\frac{\mathrm{D}\sigma_i}{\mathrm{D}t}$

teal = blue + green

violet = blue + red

magenta = blue + green + red

2. Triple-correlation transport

3. Pressure transport

4. Pressure scrambling

5. Shear production/inter-scale transfer

6. Buoyancy production

100m BOMEX, $\ell_{\mathrm{f}} = 200\ \mathrm{m}\ (= 2\Delta x)$

Condition 1: $wb > 0$ AND $w > 0$ Condition 2: $wb > 0$ AND $w < 0$ Condition 3: $wb < 0$

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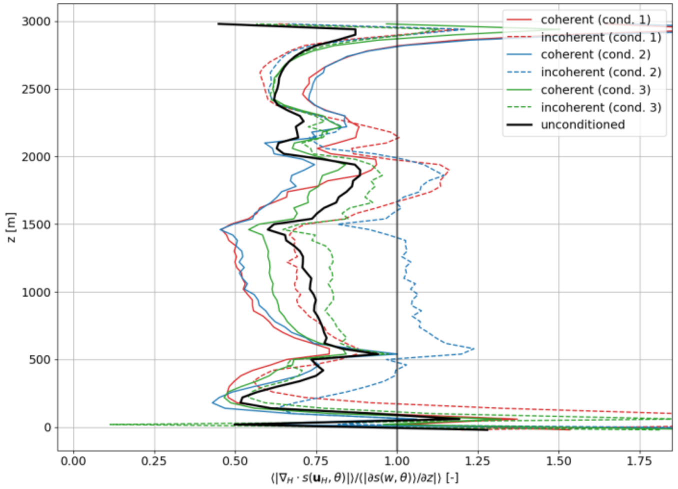
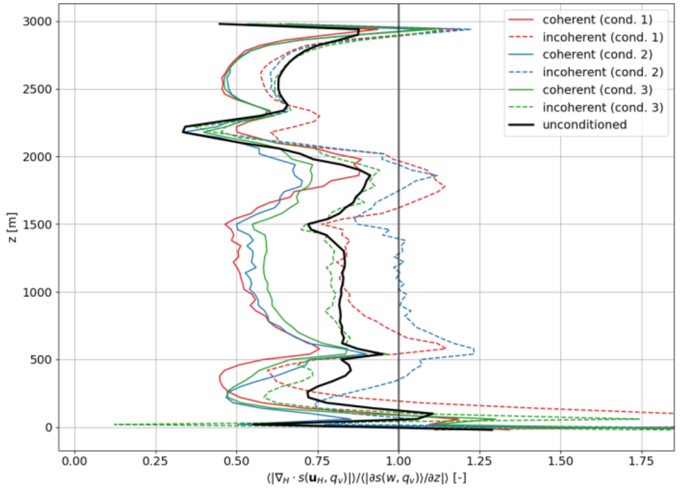
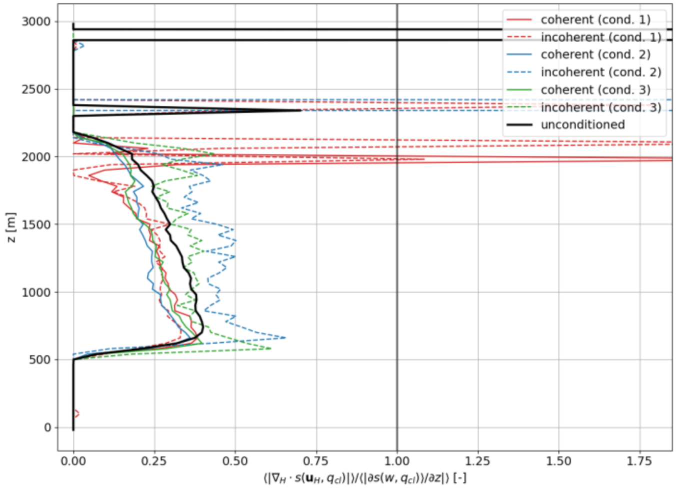
$s(\mathbf{u}, \theta)$



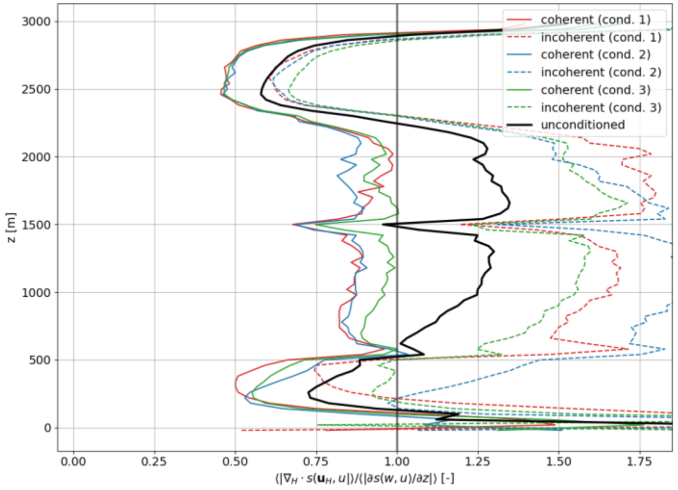
$s(\mathbf{u}, q_{\mathrm{v}})$



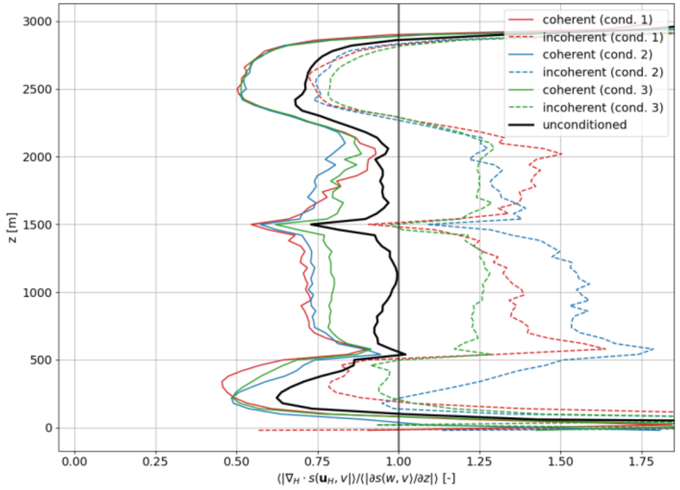
$s(\mathbf{u}, q_{\mathrm{cl}})$



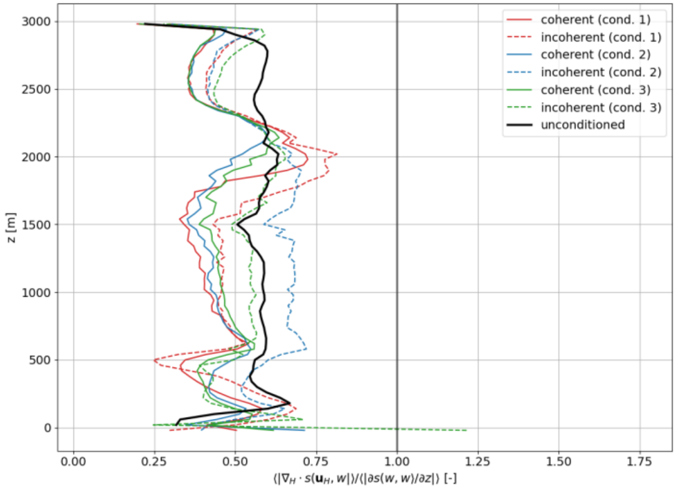
$s(\mathbf{u}, u)$



$s(\mathbf{u}, v)$

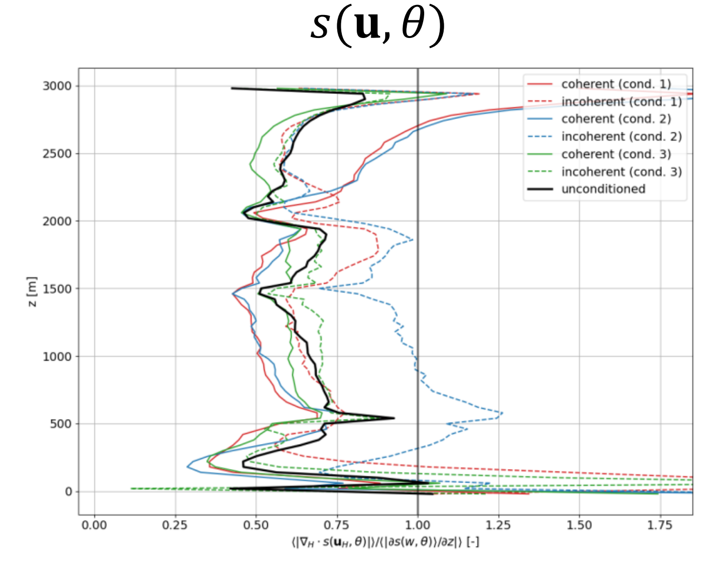


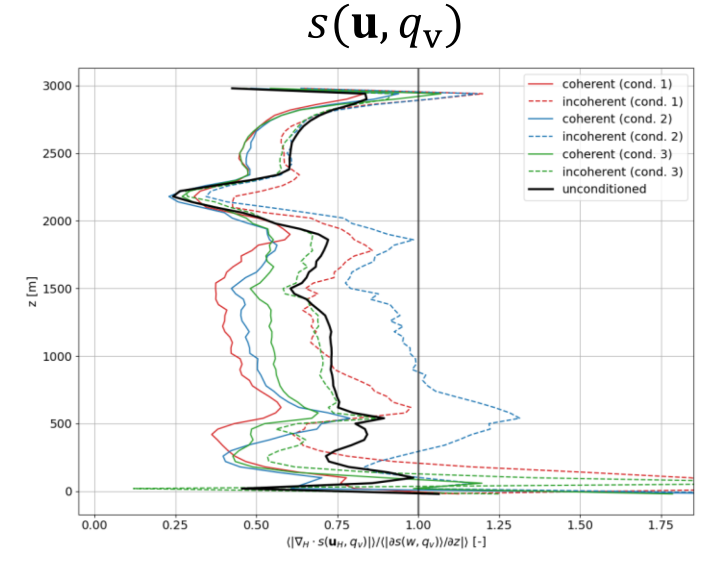
$s(\mathbf{u}, w)$

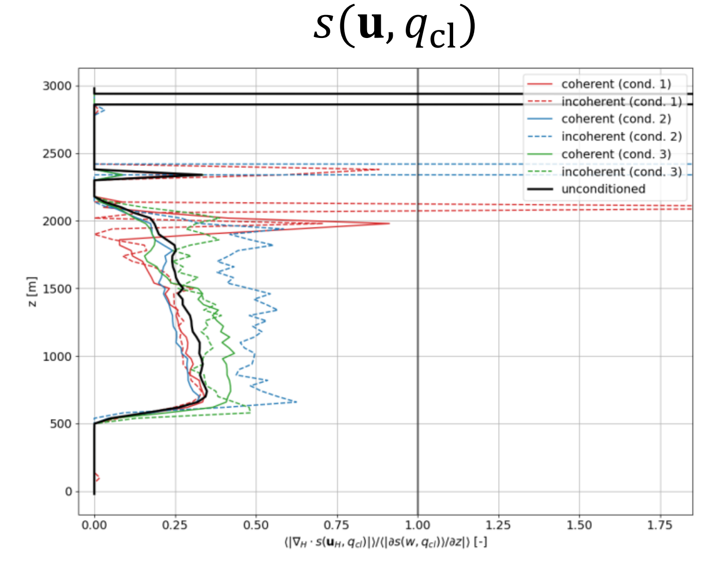


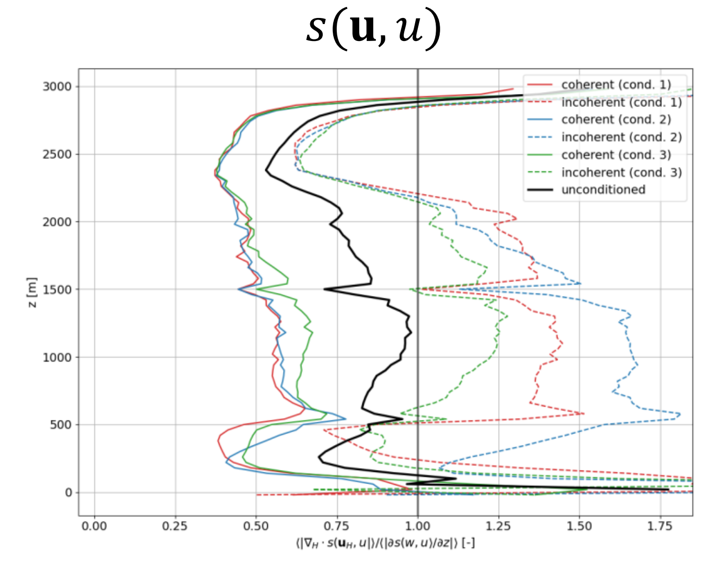
100m BOMEX, $\ell_{\mathrm{f}} = 400\ \mathrm{m}\ (= 4\Delta x)$

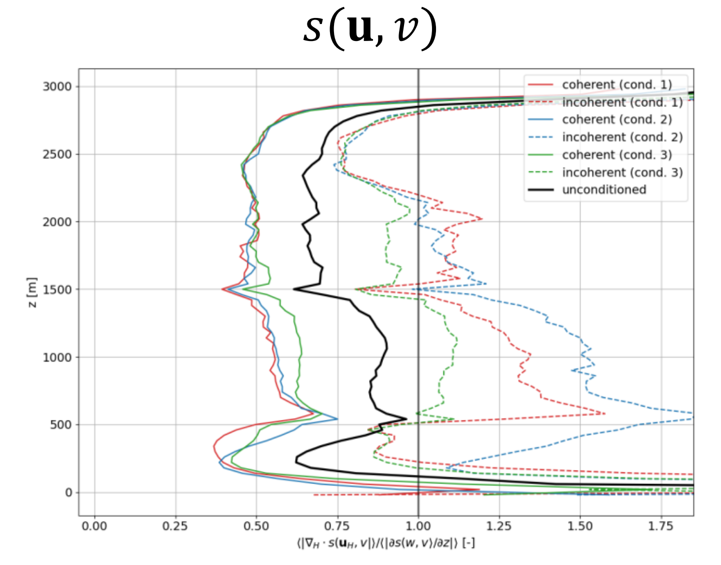
Condition 1: $wb > 0$ AND $w > 0$ Condition 2: $wb > 0$ AND $w < 0$ Condition 3: $wb < 0$

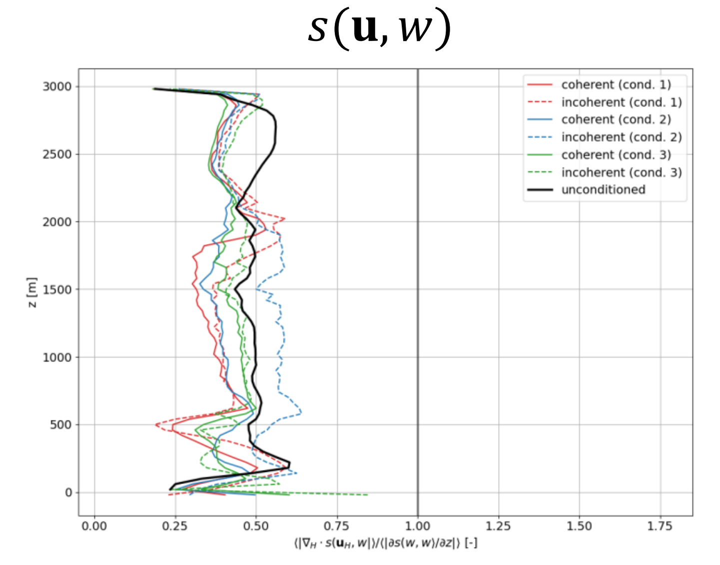

$s(\mathbf{u}, \theta)$


$s(\mathbf{u}, q_{\mathrm{v}})$


$s(\mathbf{u}, q_{\mathrm{cl}})$


$s(\mathbf{u}, u)$


$s(\mathbf{u}, v)$


$s(\mathbf{u}, w)$

100m BOMEX, $\ell_\mathrm{f} = 1000\ \mathrm{m}\ (= 10\Delta x)$ Condition 1: $wb > 0$ AND $w > 0$ Condition 2: $wb > 0$ AND $w < 0$ Condition 3: $wb < 0$

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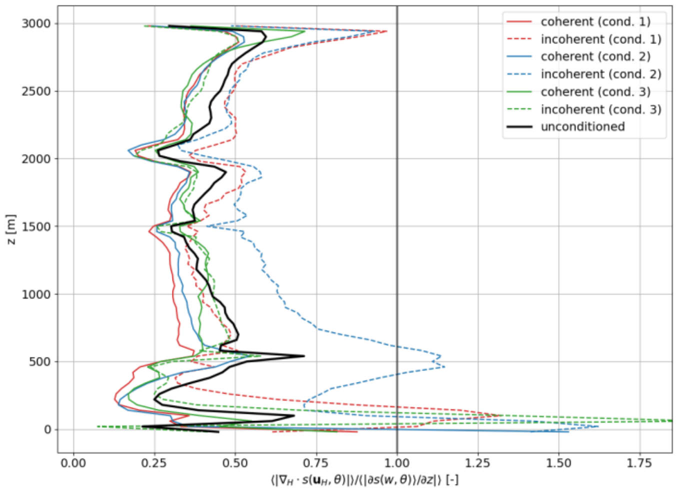
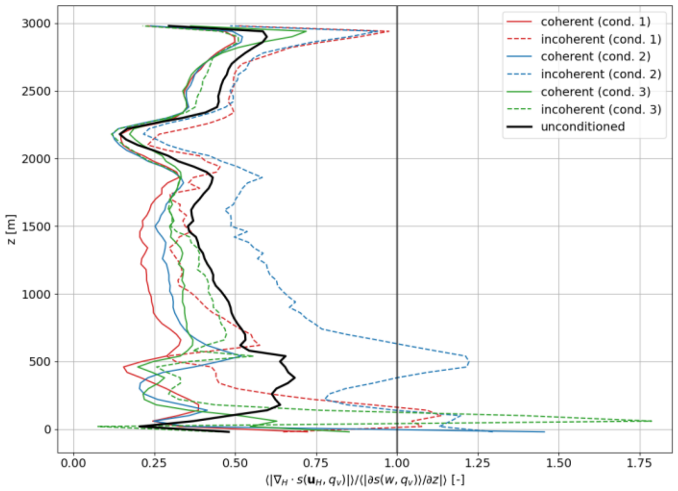
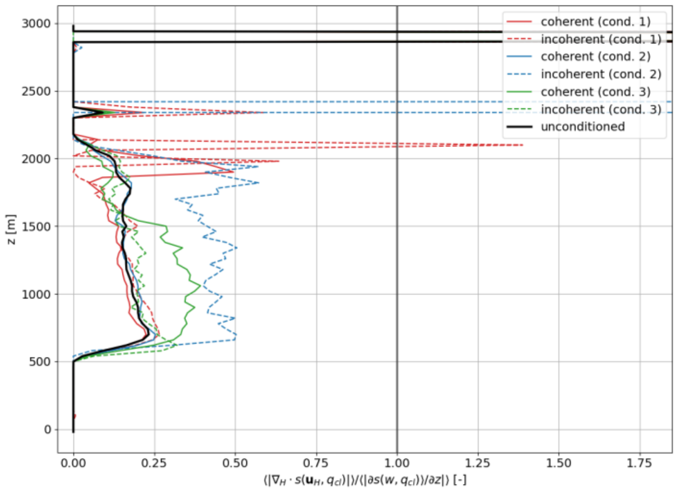
$s(\mathbf{u}, \theta)$



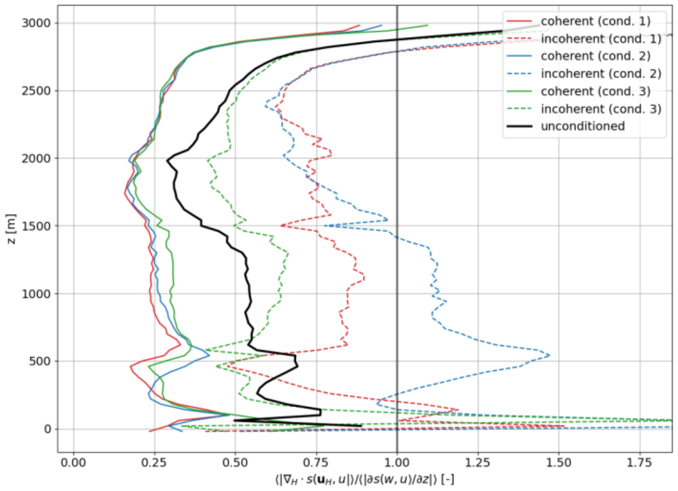
$s(\mathbf{u}, q_\mathrm{v})$



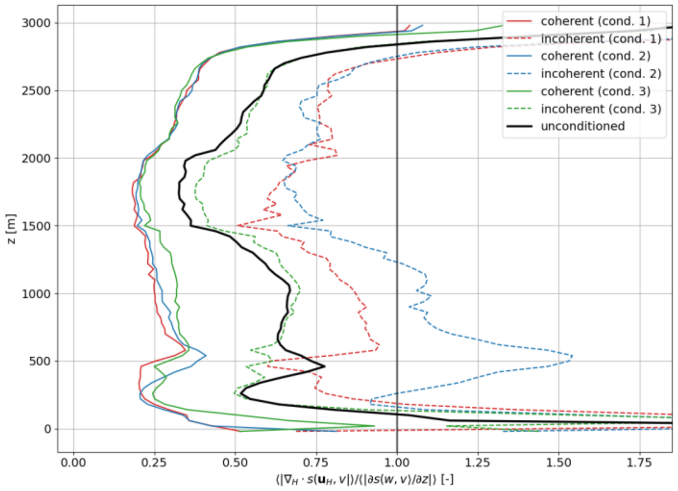
$s(\mathbf{u}, q_\mathrm{cl})$



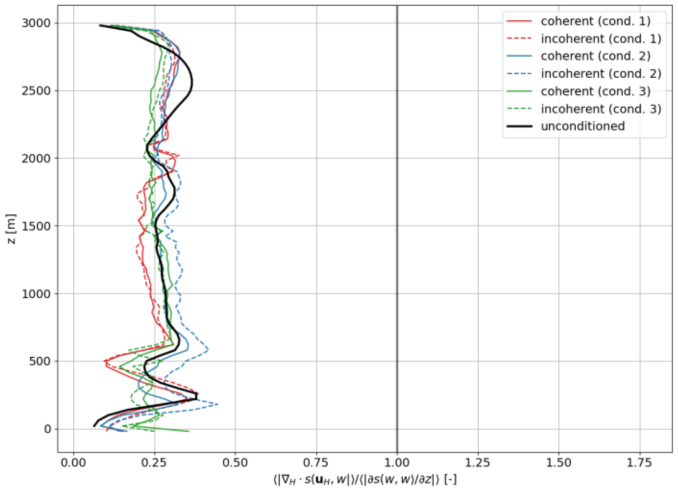
$s(\mathbf{u}, u)$



$s(\mathbf{u}, v)$

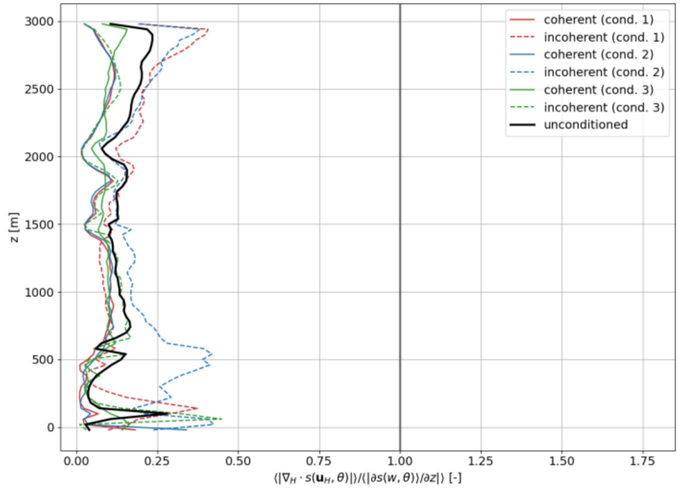


$s(\mathbf{u}, w)$

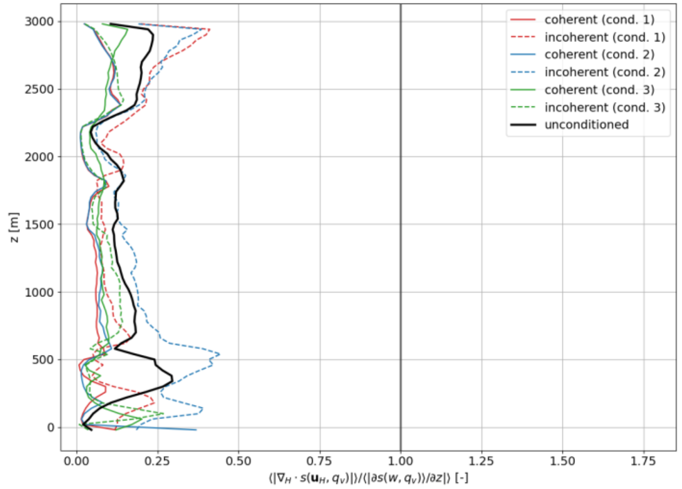


100m BOMEX, $\ell_\mathrm{f} = 4000\ \mathrm{m}\ (= 40\Delta x)$ Condition 1: $wb > 0$ AND $w > 0$ Condition 2: $wb > 0$ AND $w < 0$ Condition 3: $wb < 0$

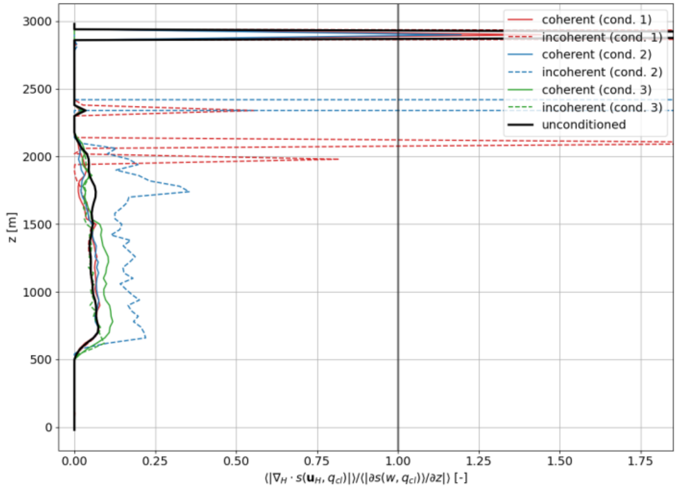
$s(\mathbf{u}, \theta)$



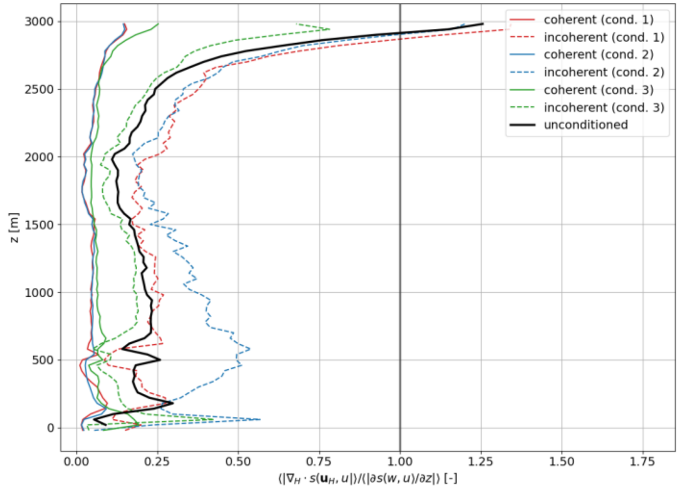
$s(\mathbf{u}, q_\mathrm{v})$



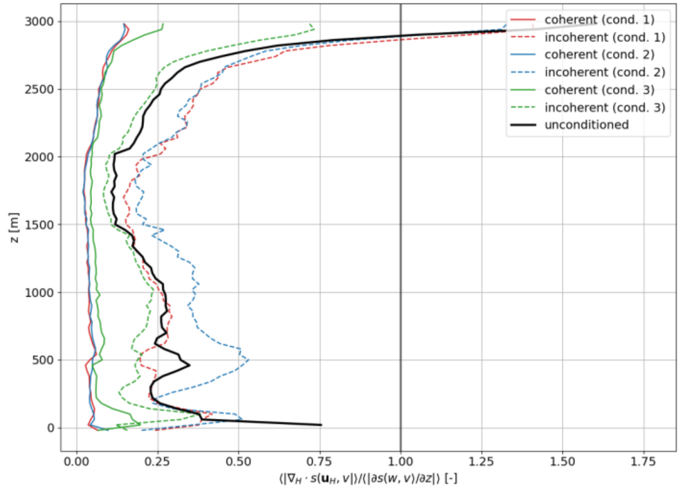
$s(\mathbf{u}, q_\mathrm{cl})$



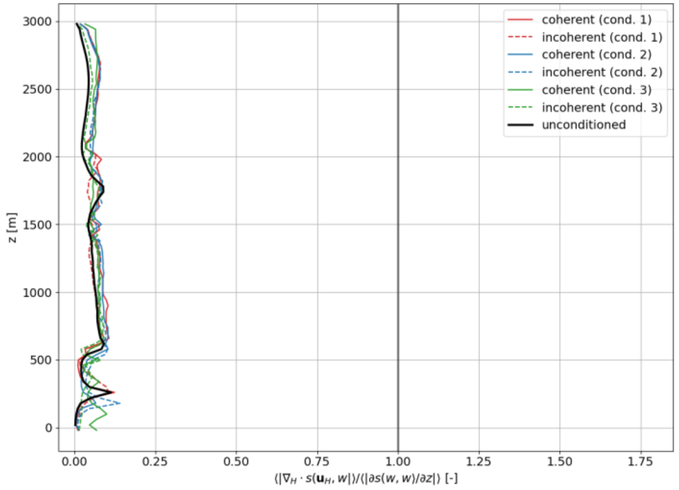
$s(\mathbf{u}, u)$



$s(\mathbf{u}, v)$



$s(\mathbf{u}, w)$



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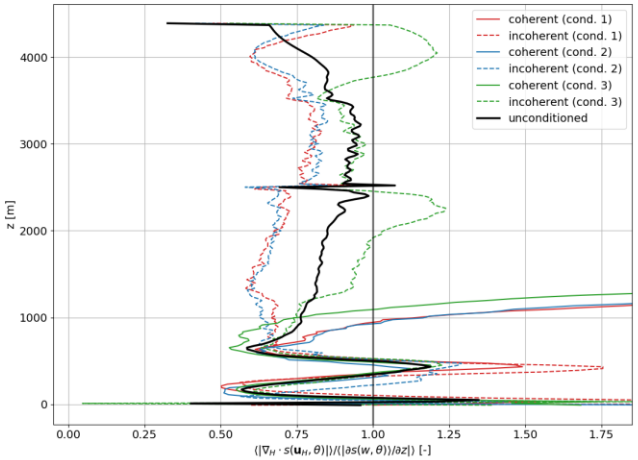
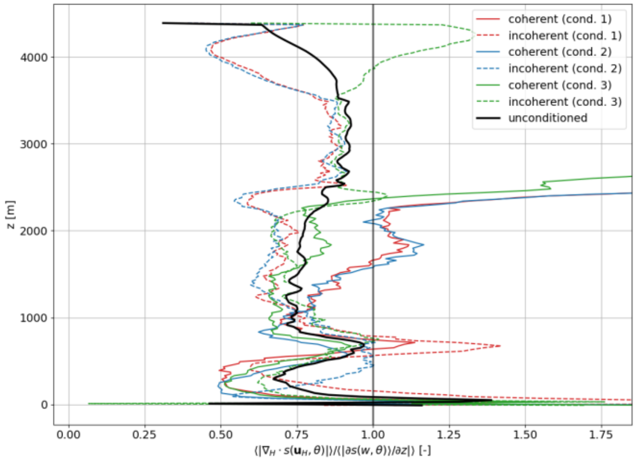
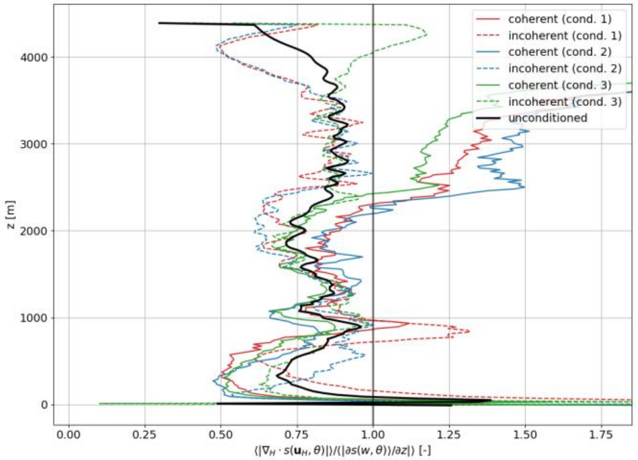
ARM_050m at t=10740.0s, filter=filter_ga0025, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$



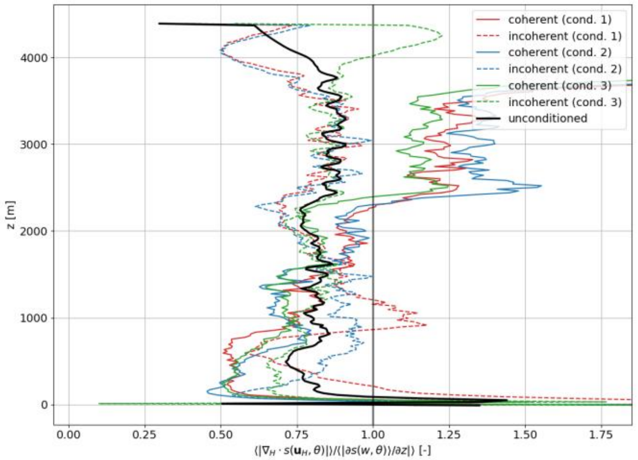
ARM_050m at t=14340.0s, filter=filter_ga0025, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$



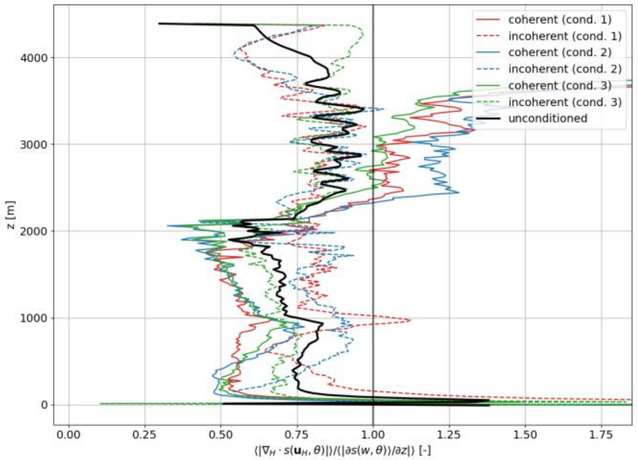
ARM_050m at t=17940.0s, filter=filter_ga0025, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$



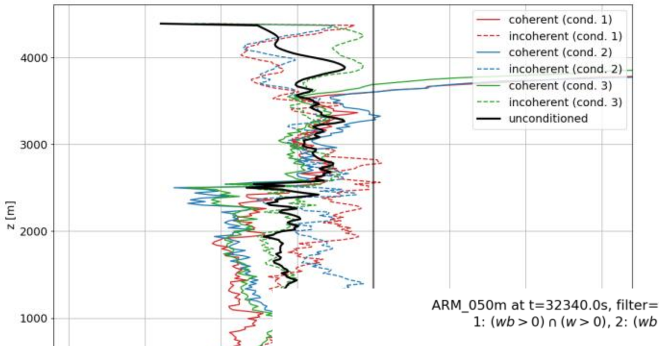
ARM_050m at t=21540.0s, filter=filter_ga0025, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$



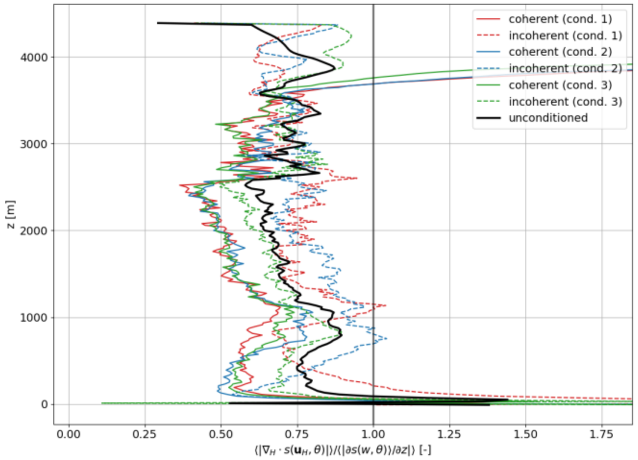
ARM_050m at t=25140.0s, filter=filter_ga0025, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$



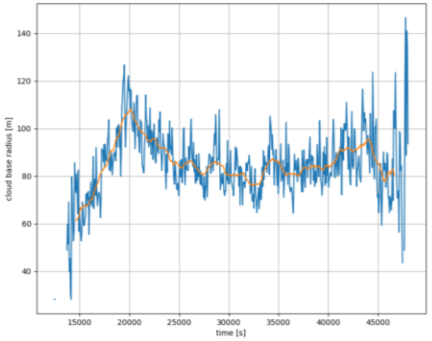
ARM_050m at t=28740.0s, filter=filter_ga0025, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$

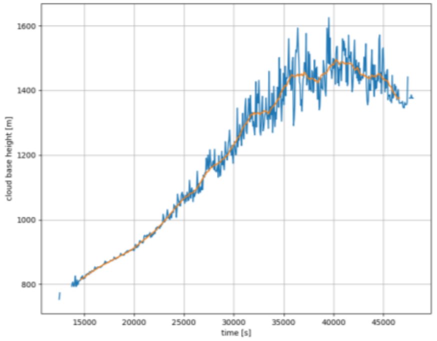


ARM_050m at t=32340.0s, filter=filter_ga0025, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$



100 m





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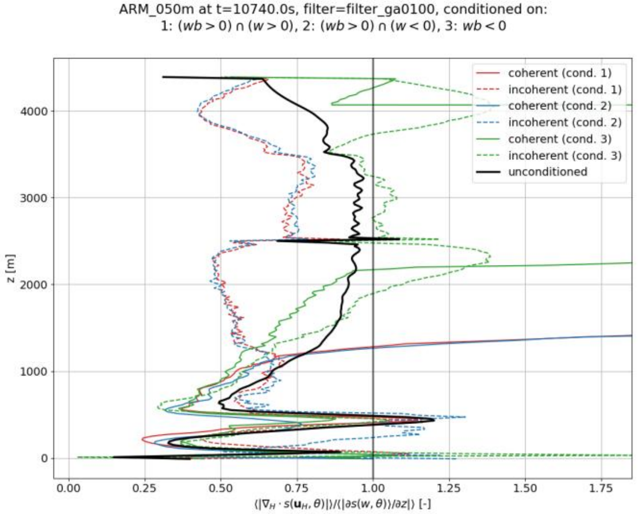
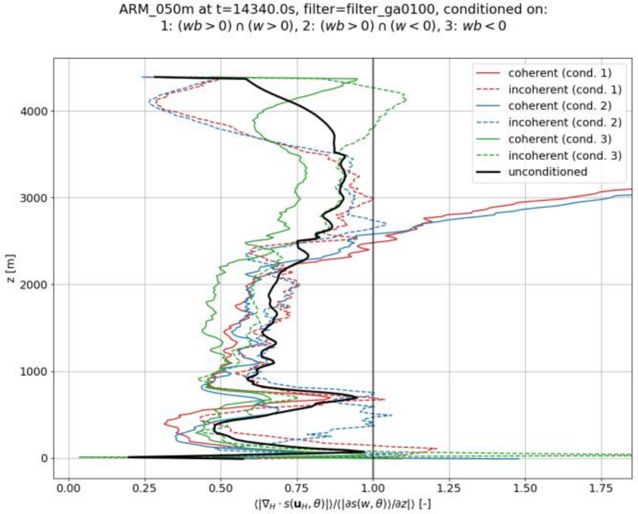
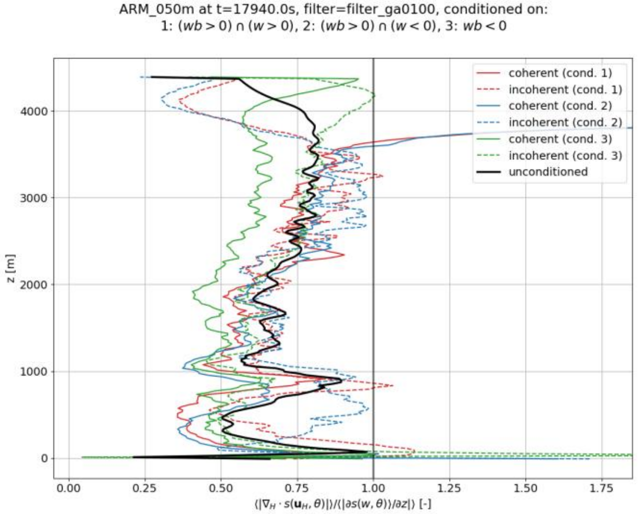
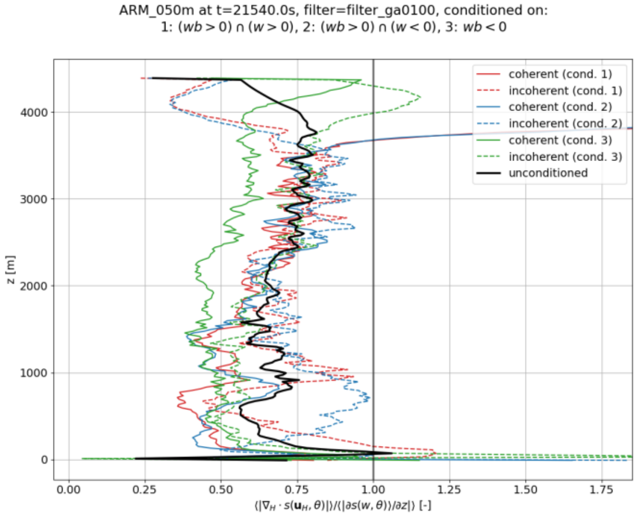
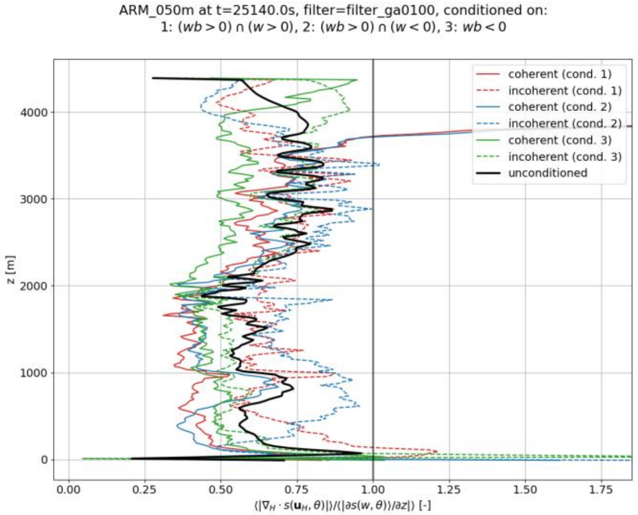
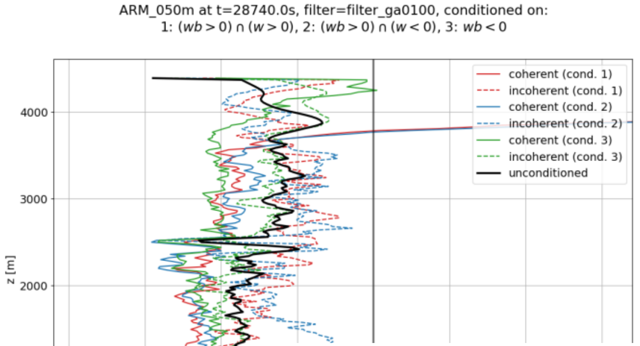

ARM_050m at t=10740.0s, filter=filter_ga0100, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$

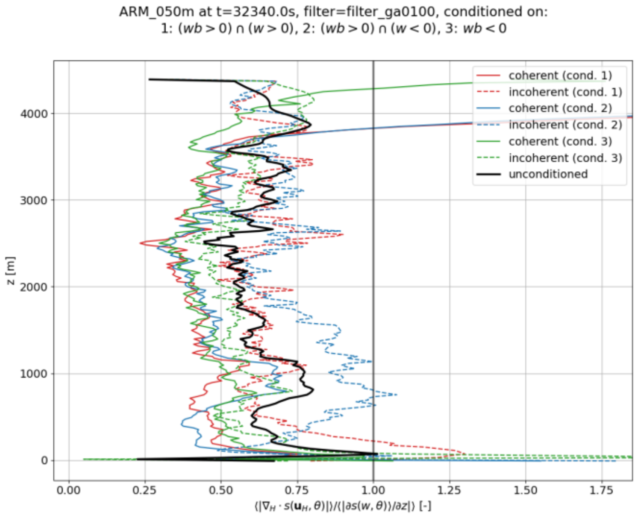

ARM_050m at t=14340.0s, filter=filter_ga0100, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$


ARM_050m at t=17940.0s, filter=filter_ga0100, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$

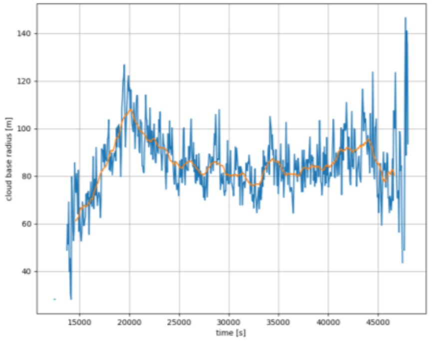

ARM_050m at t=21540.0s, filter=filter_ga0100, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$

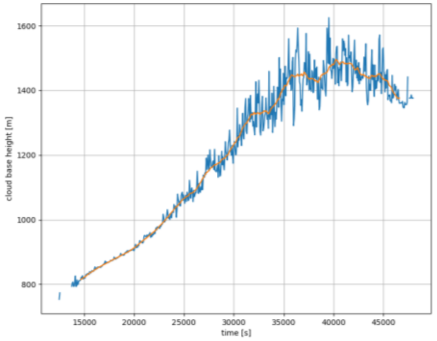

ARM_050m at t=25140.0s, filter=filter_ga0100, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$


ARM_050m at t=28740.0s, filter=filter_ga0100, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$

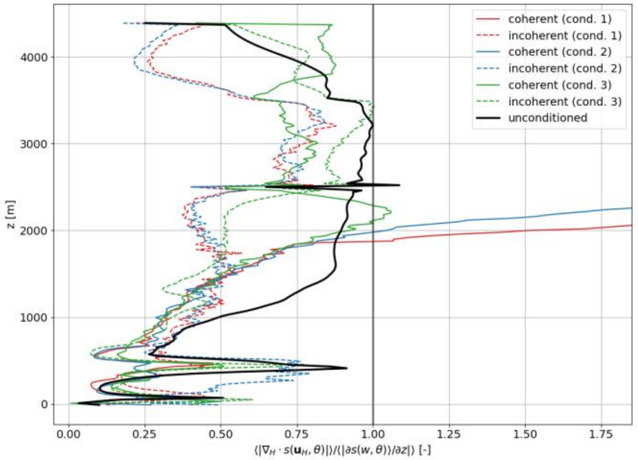

ARM_050m at t=32340.0s, filter=filter_ga0100, conditioned on:
1: $(wb > 0) \cap (w > 0)$, 2: $(wb > 0) \cap (w < 0)$, 3: $wb < 0$

400 m

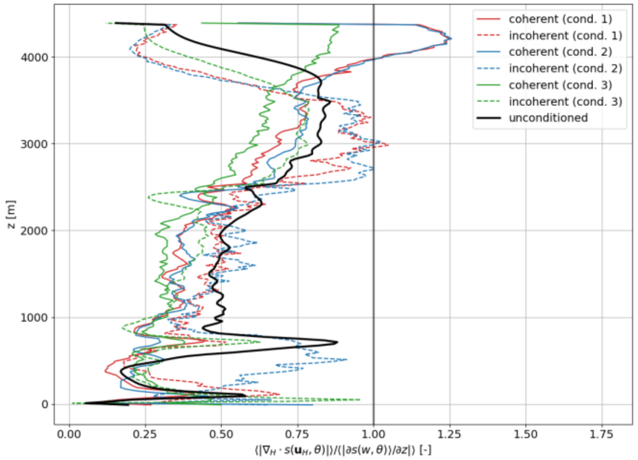




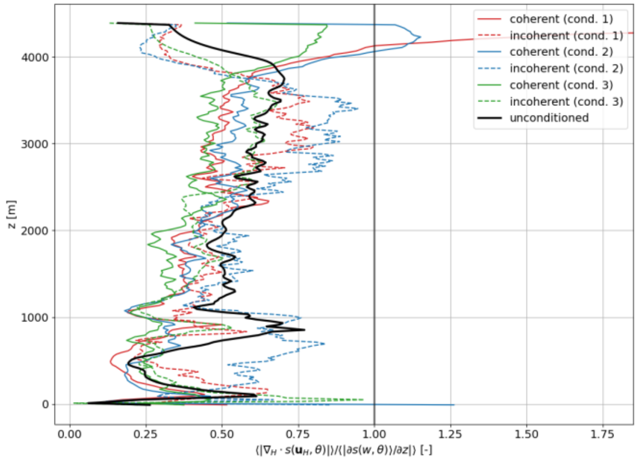
ARM_050m at t=10740.0s, filter=filter_ga0250, conditioned on:
1: $(wb>0)\cap(w>0)$, 2: $(wb>0)\cap(w<0)$, 3: $wb<0$



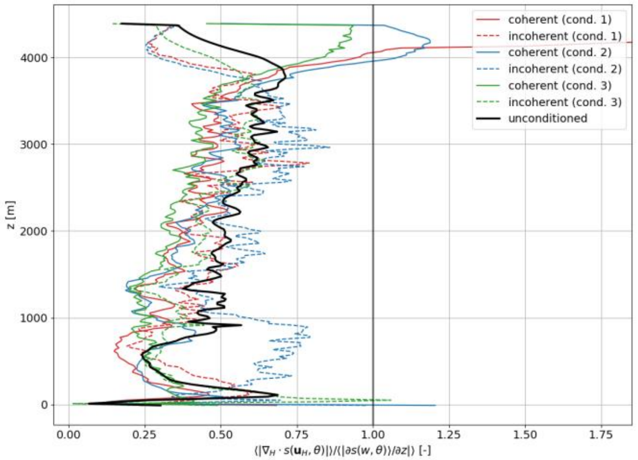
ARM_050m at t=14340.0s, filter=filter_ga0250, conditioned on:
1: $(wb>0)\cap(w>0)$, 2: $(wb>0)\cap(w<0)$, 3: $wb<0$



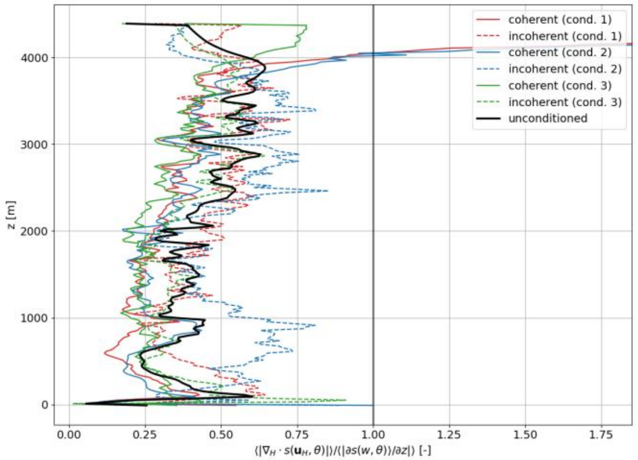
ARM_050m at t=17940.0s, filter=filter_ga0250, conditioned on:
1: $(wb>0)\cap(w>0)$, 2: $(wb>0)\cap(w<0)$, 3: $wb<0$



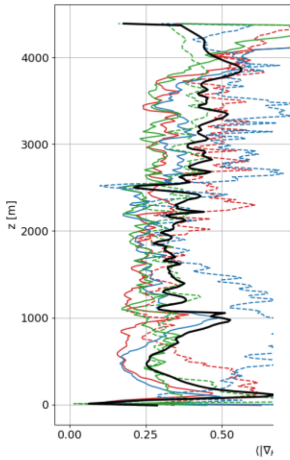
ARM_050m at t=21540.0s, filter=filter_ga0250, conditioned on:
1: $(wb>0)\cap(w>0)$, 2: $(wb>0)\cap(w<0)$, 3: $wb<0$



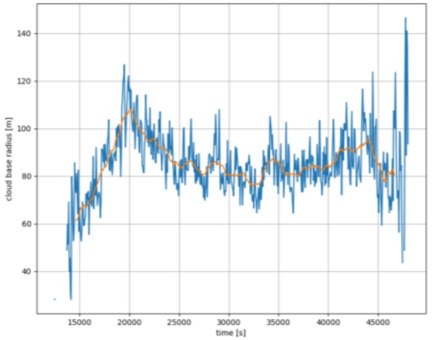
ARM_050m at t=25140.0s, filter=filter_ga0250, conditioned on:
1: $(wb>0)\cap(w>0)$, 2: $(wb>0)\cap(w<0)$, 3: $wb<0$

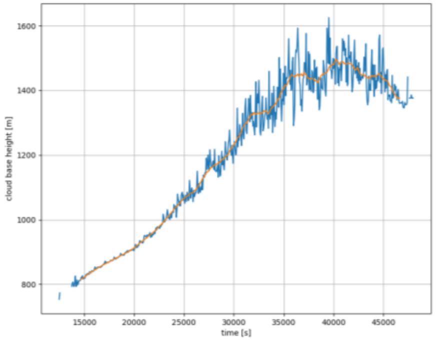


ARM_050m at t=28740.0s, filter=filter_ga0250, conditioned on:
1: $(wb>0)\cap(w>0)$, 2: $(wb>0)\cap(w<0)$, 3: $wb<0$

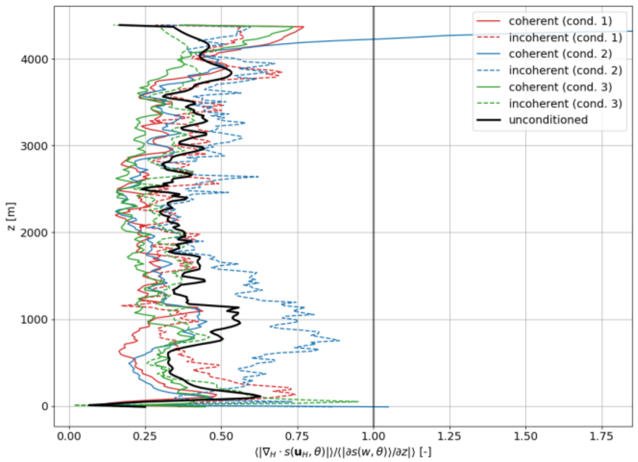


1000 m





ARM_050m at t=32340.0s, filter=filter_ga0250, conditioned on:
1: $(wb>0)\cap(w>0)$, 2: $(wb>0)\cap(w<0)$, 3: $wb<0$



Overarching goal

WP4:

- Using s(w,w) [especially s(w,w,w) and pressure scrambling] to think about how 3DTE & CM should interface (i.e. handing over from a 3D to a 1D scheme)
- S(www) and pressure scrambling are obvious candidates for mass flux-esque modifications to 3DTE in order to smooth transition
- CoMorph prognostic s(w,w) can be derived from sum of coherent & incoherent fluxes over all partitions
- When l_inter-cloud > l_filter > l_cloud, horizontal fluxes MUST be important
- Given M, w, can get \sigma; if we have a cloud length scale (e.g. CM cloud base radius from BL diffusivity), this also implies an l_ic