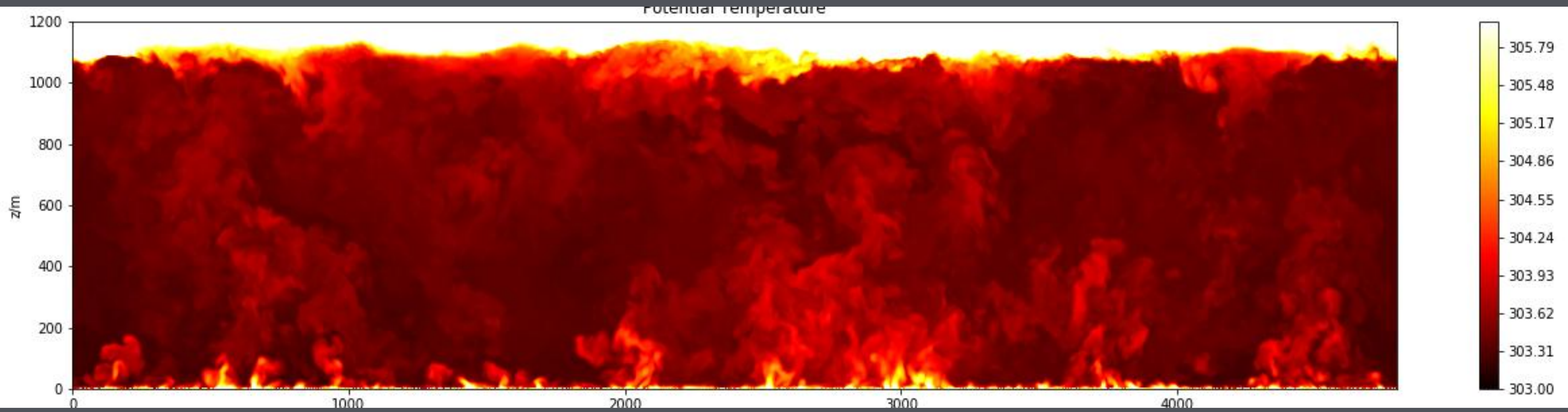
Meteorology

Implementation of dynamic filtering with grey-zone turbulent closures in a Numerical Weather Prediction Model: Evaluation in idealised cases



Yuqi Bai, Peter Clark, Dimitar Vlaykov, George Efstathiou, Bob Plant

Thanks to many.

Introduction

- Starting point: spatially filtered equations of motion and thermodynamics with ‘standard’ (for now) closures.
- HiFi is bringing together developments in the solution (3DTE) and dynamic methods for parameter estimation within the UM.
- Bringing together new ideas and developments from perhaps 30 years (or more)!
- Evaluation in turbulence ‘grey-zone’ in idealised and real (WesCon) cases.
- Idealised: CBL, BOMEX, ARM, LBA (RCE …).
    - https://code.metoffice.gov.uk/trac/rmed/ticket/551
- Here we focus on CBL as it illustrates some issues and developments.

- Notation:
    - $\phi = \phi^r + \phi^s$.
    - $s(\phi, \psi) \equiv (\phi\psi)^r - \phi^r\psi^r$.
    - $s(u_i, \psi)$ is the analogue of $\langle u'\psi' \rangle$ **but it is generally not equal to** $(u^s\psi^s)^r$.

HiFi is a project funded by the NERC project: PARACHUTE

SGS turbulence parametrization: modified Smagorinsky

Options: "The toppings"

- $\lambda \coloneqq C_s \Delta\,;\, \nu_m \approx \lambda^2|S|;\, \nu_h \approx \frac{\lambda^2|S|}{Pr}$
- Down-gradient only SGS turbulence transport: $s(u_i, u_j) \approx -\nu_m S_{ij}^r\,;\, s(\psi, u_j) \approx -\nu_h \frac{\partial \psi^r}{\partial x_j}$

Smagorinsky parameterization

"The Pizza dough"

+ Richardson number dependence (stability function)	• $\nu_m \approx \lambda^2|S|f_m(Ri)$, $\nu_h \approx \lambda^2|S|f_h(Ri)$ • Pr as a function of Ri
+ blended length scale	• Surface layer blending • Blend with BL scheme length scale (Blackadar)
+ dynamic length scale	• Dynamically parameterize length scale by dynamical filtering • (Will be covered in detail in following slides)
+ Leonard/tilting terms (mixed model)	• SGS flux from tilting in direction effects

In development

In UM already

In other published works

SGS turbulence parametrization: modified Mellor-Yamada schemes

- Schemes conserving TKE (i.e. $s(u_i, u_i)$), QSQ ($s(q_t, q_t)$), TSQ ($s(\theta_L, \theta_L)$) and COV ($s(\theta_L, q_t)$)
- Level 4: Prognostic TKE, deviatoric stress, scalar fluxes, variance and covariances, 1+5+2*3+3=15 prognostic equations to solve
- Level 3: Level 4 but diagnostic deviatoric stress and fluxes, 1+3=4 prognostic eqns.
- Level 2.5: Level 3 but diagnostic variance and covariances, i.e. TKE prognostic only
- Level 2: Diagnostic TKE, equivalent to Smagorinsky's local equilibrium assumption

Nakanishi-Niino Scheme (MYNN)

- 1D BL scheme
- 1D Ri, gradients, fluxes

3DTE schemes (Mk1 & Mk2)

- 3D turbulence scheme
- 3D Ri, gradients, fluxes
- Please see P.Clark's presentation for more details

3DTE: The Full (approximate) Solution

- Full (closed but un-approximated) prognostic equations for:
  - TKE (or $u_t^2 = s(u_k, u_k)$)
  - $s(\theta_L, \theta_L), s(q_t, q_t)$ and $s(\theta_L, q_t)$ from which we obtain $s(\phi, b)$ for any scalar.

LEVEL 3:

- Approximate solution to local steady-state stress and scalar fluxes.
  - Three terms: down-gradient, counter-gradient and shear production/tilting.
  - The last has the same form as the 'Leonard-term' parametrization.

(Approximate) solution for 3D scalar fluxes:

$$s(u_i, \phi) = -Lu_t \left[S_H \frac{\partial \phi^r}{\partial x_i} + \Gamma_\phi \delta_{i3} \right] + S_H' L^2 \frac{\partial u_i^r}{\partial x_k} \frac{\partial \phi^r}{\partial x_k}$$

Down-gradient | Counter-gradient | Shear Production/ Tilting/Leonard

LEVEL 2:

Diagnostic forms lead to

$$u_t = L|S| f_{u_t}(Ri)$$

$$Lu_t \Gamma_\phi = -L^2 S_H'' \frac{\frac{\partial \phi^r}{\partial x_k} \frac{\partial b^r}{\partial x_k}}{|S|}$$

3DTE: Mk I and Mk II

- Mk I:
  - Make 1D Nakanishi-Niino Mellor-Yamada scheme use 3D shear where appropriate.
  - Blended mixing length asymptotes to $C_s\Delta$.
  - Use vertical viscosity/diffusivity in horizontal.
  - Leonard term approximates tilting.
  - Optionally provide Leonard Term with coefficient.
- Mk II:
  - Theoretically more rigorous. (Some unavoidable inconsistencies avoided).
  - Complete 3D scheme written from scratch (following Mk I structure/variables).
  - Effectively moves some ‘vertical diffusion’ into ‘tilting’.

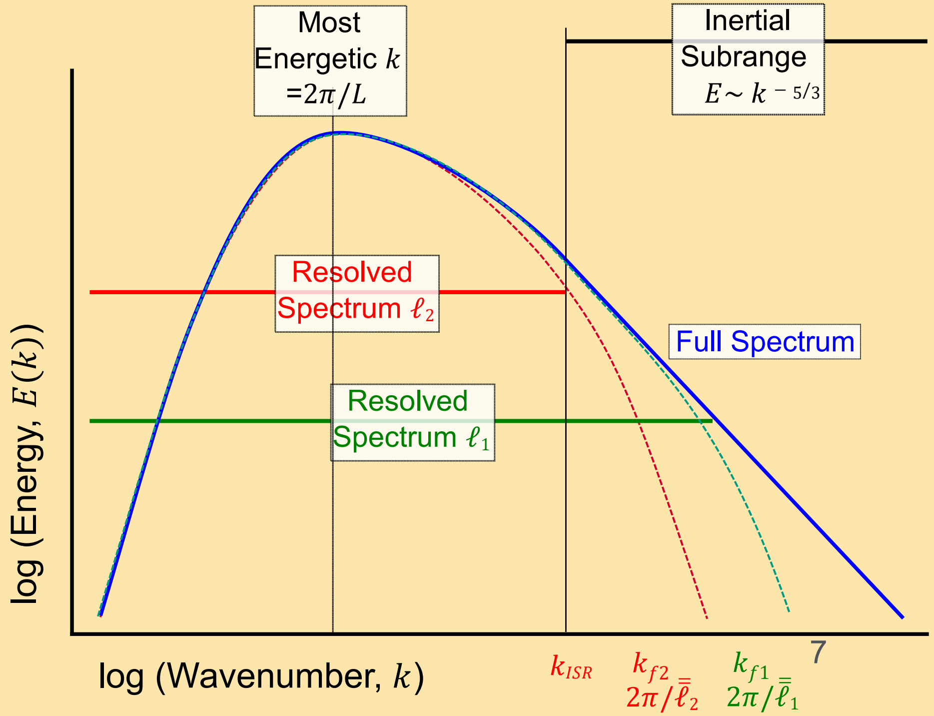
Dynamic filtering

Rationale (over-simplified):

1. We wish our parametrization for sub-grid momentum flux is less scale-dependent (works better in grey-zone)
2. Find one spatial test filter the length scale of which is larger than (usually 2 times) the grid scale
3. From the Germano identity (see next slide), the difference (L_{ij}) between sub-test-filter momentum flux (T_{ij}) and sub-grid-filter momentum flux (τ_{ij}) filtered , should equals a known filtered field of resolved scale velocity field, using same test filter
4. We modify our closure constants in parametrization as a function of length scale, so that the parametrization do better in both sub-test filter and sub-grid filter momentum flux
5. Justify by comparing the **parametrized** difference and the **filtered** difference (L_{ij})

Options:

- More than 1 test filters: “test-of-test” filter
- More than dynamical momentum-flux (depend on formulation): may also dynamical scalar fluxes / Pr number



Germano identity and Lilly minimisation

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- between a generalised larger (superscript r) and smaller (overbar) filter scale, with length scale of $\boldsymbol{\Lambda}$ and $\boldsymbol{\lambda}$ respectively

If we have a parametrization of sub-filter scale L stress for arbitrary filter length scale L:

$$s(u_i, u_j)\Big|_L = f_{param}(L)$$

Sub "overbar" scale stress, Filtered onto "superscript r" scale

$$-(\tau_{ij})^r := -\left(\left(\overline{u_i u_j}\right)^r - \left(\bar{u}_i \bar{u}_j\right)^r\right)$$

+

Sub "superscript r" scale stress

$$\mathrm{T}_{ij} := \left(\overline{u_i u_j}\right)^r - \left(\bar{u}_i\right)^r \left(\bar{u}_j\right)^r$$

=

Stress between the two scales, filtered onto "superscript r" scale

$$L_{ij} \equiv \left(\bar{u}_i \bar{u}_j\right)^r - \left(\bar{u}_i\right)^r \left(\bar{u}_j\right)^r$$

(Known from filtering the fields)

Then:

$$-\left(f_{param}(\lambda)\right)^r + f_{param}(\Lambda)$$

Should tend to

By changing f_{params} !

Dynamic Smagorinsky

Smagorinsky with stability functions gives for filter length L and filter $(x)^R$ with length scale L :

$$s(u_i,u_j)^R\Big|_L = f_{param}(L) = -\nu_m(S_{ij})^R = -L^2|S|^R S_{ij}^R f_m((Ri)^R)$$

And $L = C_{s,L}\Delta$

Then for grid filter (overbar, λ) and one test filter (superscript r, Λ):

$L_{ij} \equiv (\overline{u_i}\,\overline{u_j})^r - (\overline{u_i})^r(\overline{u_j})^r$ Should tend to $-\left(f_{param}(\lambda)\right)^r + f_{param}(\Lambda)$

where $-\left(f_{param}(\lambda)\right)^r = \left(\lambda^2|\bar{S}|\bar{S}_{ij}f_m(\overline{Ri})\right)^r$ and $f_{param}(\Lambda) = -\Lambda^2|\bar{S}|^r\bar{S}_{ij}^r f_m((\overline{Ri})^r)$

$$= \Delta^2\left[\left(C_{s,\lambda}^2|\bar{S}|\bar{S}_{ij}f_m(\overline{Ri})\right)^r - \left(\frac{\Lambda}{\lambda}\right)^2 C_{s,\Lambda}^2|\bar{S}|^r\bar{S}_{ij}^r f_m((\overline{Ri})^r)\right]$$

Now use $\Lambda = 2\lambda$ and assume $\dfrac{C_{s,\Lambda}^2}{C_{s,\lambda}^2} = \beta$ then:

$$L_{ij} \text{ Should tend to } \underbrace{\Delta^2\left[\left(|\bar{S}|\bar{S}_{ij}f_m(\overline{Ri})\right)^r - 4\beta|\bar{S}|^r\bar{S}_{ij}^r f_m((\overline{Ri})^r)\right]}_{M_{ij}} C_{s,\lambda}^2$$

If β known a priori:

$$MINIMISE\left\{\sum\left[L_{ij} - C_{s,\lambda}^2 M_{ij}\right]^2\right\}$$

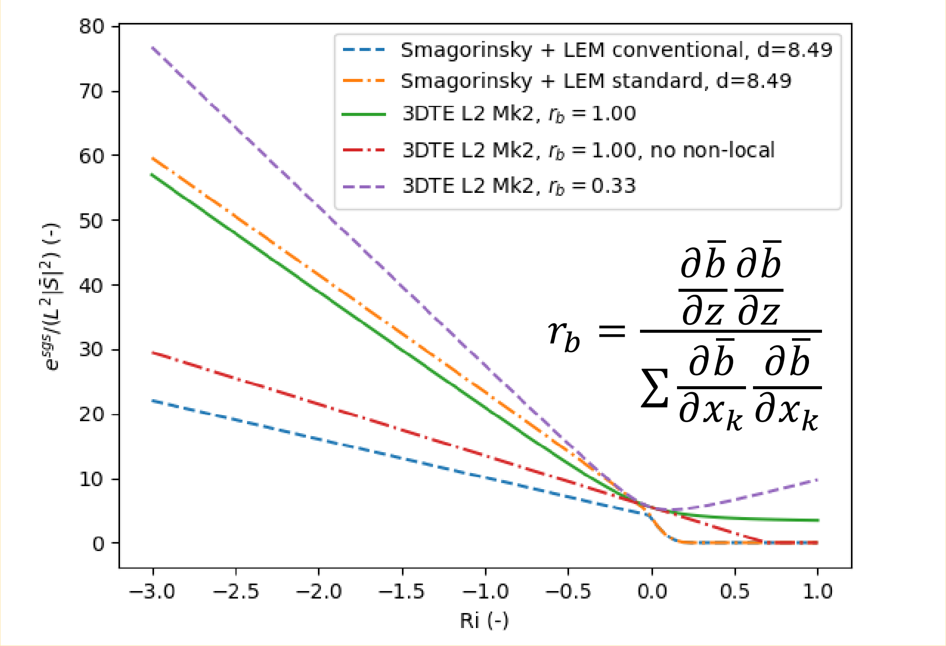
A dynamic L_{smag}!

Dynamic 3DTE L2

- All-diagnostic equations for stress and TKE enables analogy to Smagorinsky
- If turn off counter-gradient terms and tilting / Leonard / mixed-model terms, 3DTE L2 should be consistent with Smagorinsky only with different stability functions (in good progress)
- Future works: for Dynamic 3DTE L2 with full counter-gradient and Leonard terms, since:

$$s(u_i, \phi) = L^2 \left(-S_H |S| \frac{\partial \phi^r}{\partial x_i} + S_H'' \frac{\frac{\partial \phi^r}{\partial x_k} \frac{\partial b^r}{\partial x_k}}{|S|} \delta_{i3} + S_H' \frac{\partial u_i^r}{\partial x_k} \frac{\partial \phi^r}{\partial x_k} \right)$$

$$= L^2 F(\nabla \mathbf{u}, \nabla \phi, \nabla b)$$



Dynamic method can be applied to obtain L^2 exactly as per Smagorinsky – just additional terms to compute and filter.

The Effect of Mesh Resolution on Convective Boundary Layer Statistics and Structures Generated by Large-Eddy Simulation

PETER P. SULLIVAN AND EDWARD G. PATTON

National Center for Atmospheric Research, Boulder, Colorado

Well-mixed with weak positive gradient in top half of CBL.

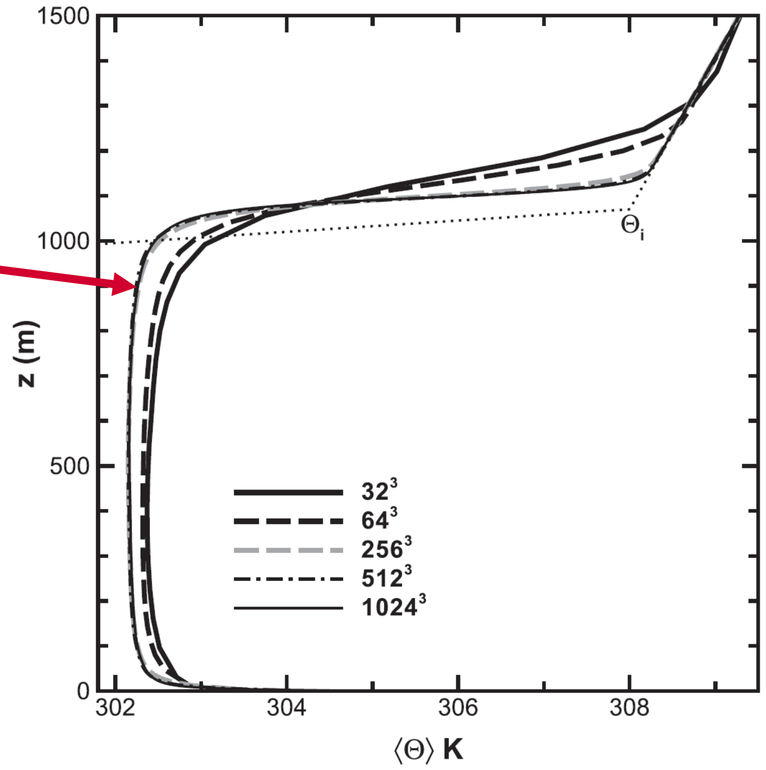


FIG. 2. Vertical profile of virtual potential temperature $\langle\overline{\theta}\rangle$ for varying mesh resolution. Note all simulations are started with the same three-layer structure for virtual potential temperature θ_i, indicated by the dotted line.

Entrainment flux

160 m x 160 m x 64 m grid box.

5 m x 5 m x 2 m grid box.

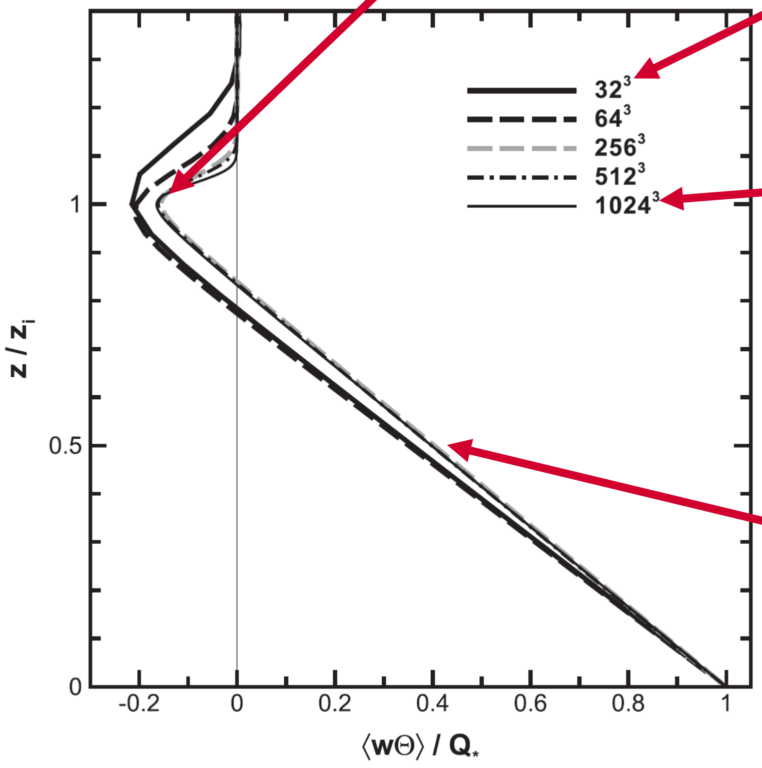
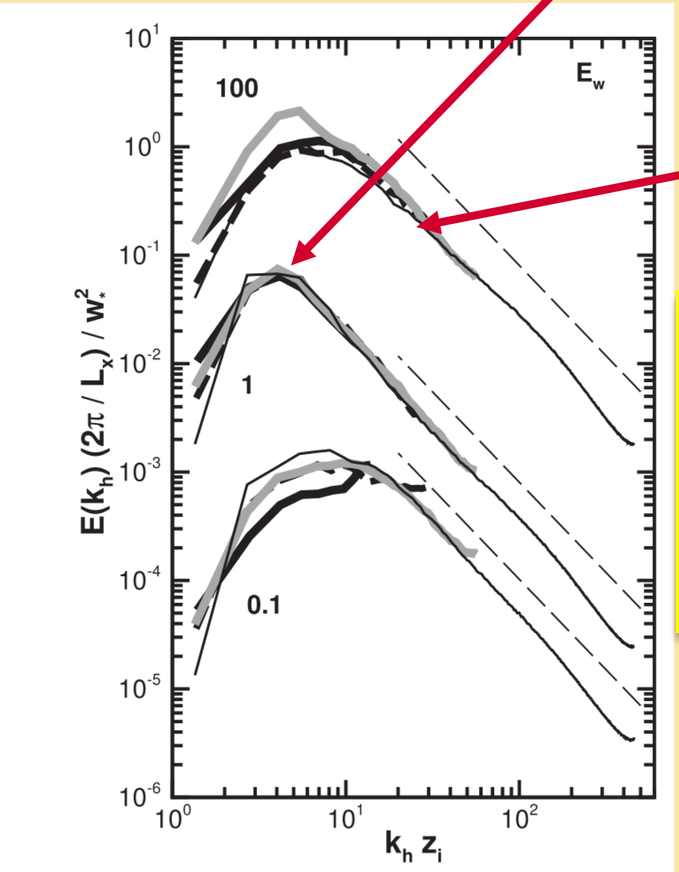


FIG. 3. Vertical profile of total temperature flux $\langle\overline{w''\theta''} + \mathbf{B}\cdot\hat{\mathbf{k}}\rangle/Q_*$ for varying mesh resolution.

Vertical heat flux follows well-known linear profile consistent with ‘uniform’ heating rate in CBL.

Peak wavenumber, i.e. most energetic wave number
$k_h z_i \sim 5$, since $z_i \sim$ 1km, $\lambda_h \sim \frac{2\pi}{k_h} \sim 1$ to 1.5 km

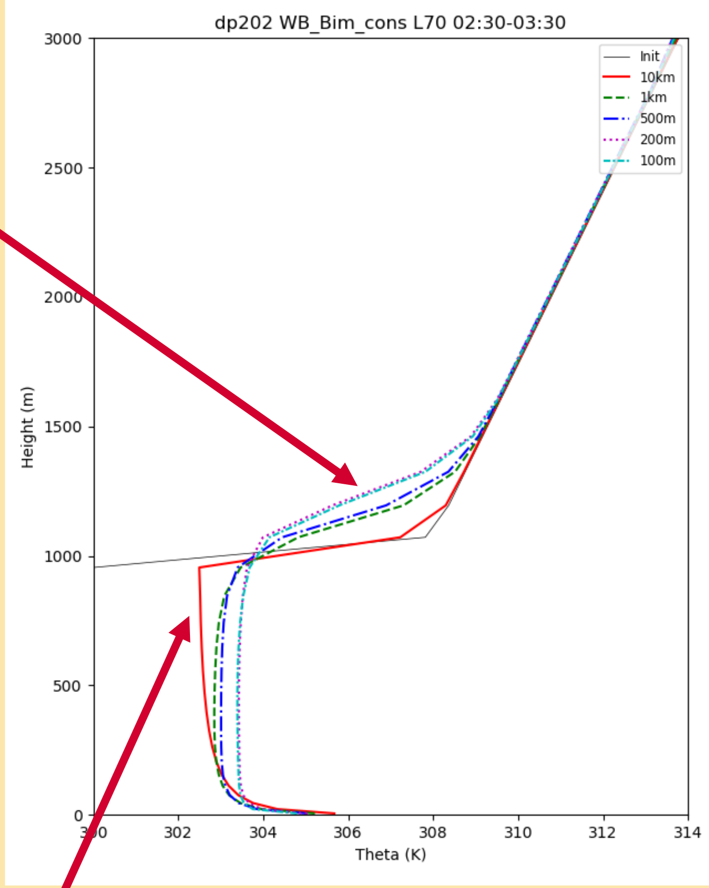


Well resolved ISR: proves that this is a "proper" LES (rather than grey-zone)

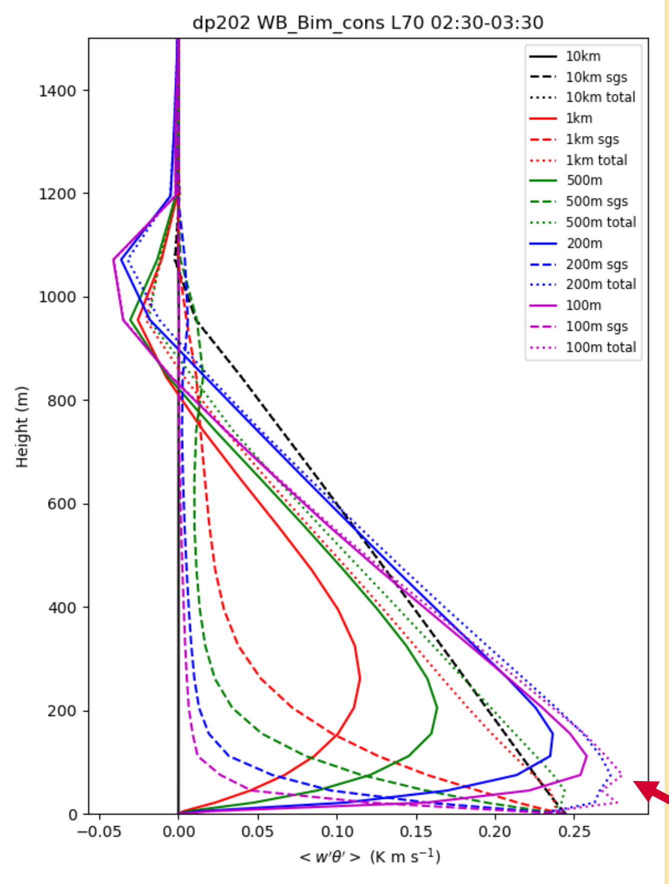
Hence if horizontal resolution >= 1km, the resolved scale turbulence should be suppressed, otherwise we get no segment of ISR
Nyquist frequency gives that we need $\Delta \leq \lambda_h/2$ to represent eddies at λ_h

Smagorinsky-Lilly in UM

Too deep, esp. at 100 m



10 km : all flux is 'sub-grid-scale' Must maintain –ve gradient



'UKV' 70-level set

$$\frac{1}{\lambda^2} = \frac{1}{(C_s\Delta)^2} + \frac{1}{\left(\kappa(z+z_0)\right)^2}$$

Too much resolved flux at 'high' resolution (though globally conserving with : *l_priestley_correct_thetav=.true.*).

Dynamic Smagorinsky

2 test filter scale (2Δ, 4Δ), variable Pr

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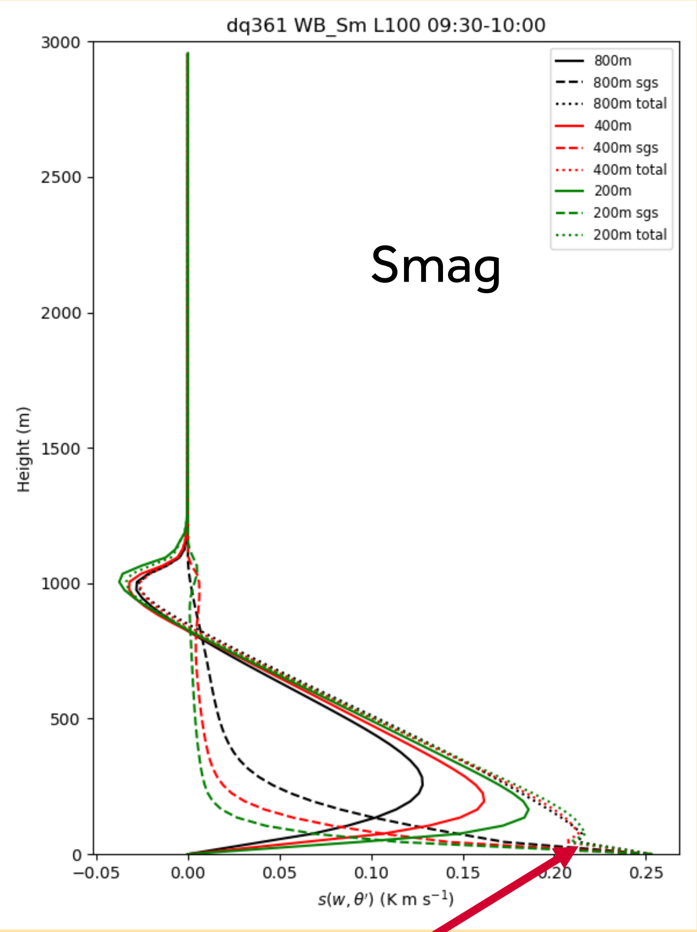
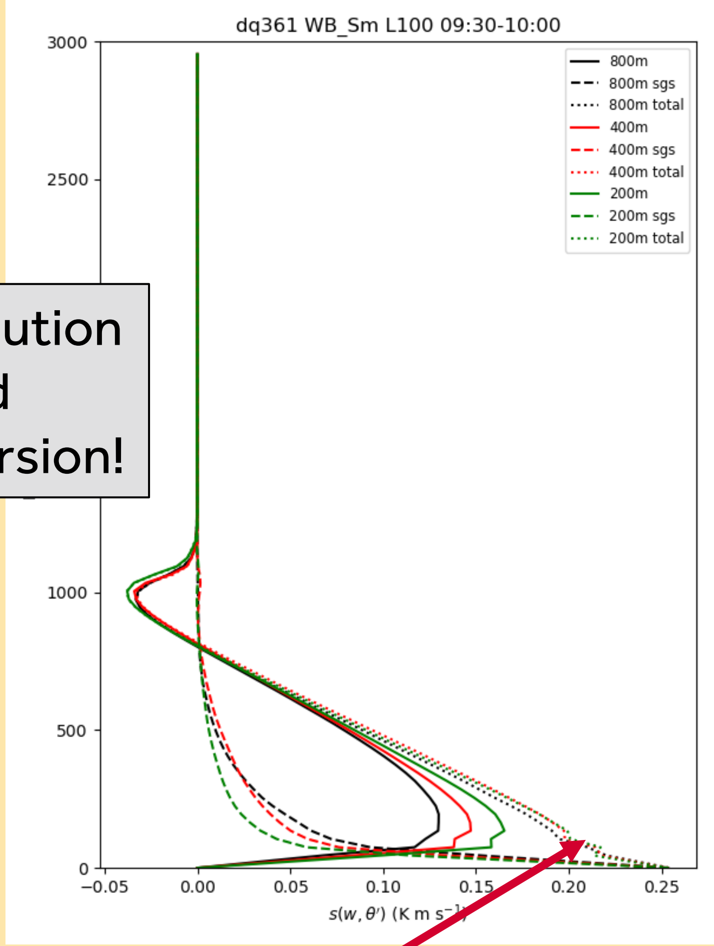
1 test filter scale (2Δ), fixed Pr





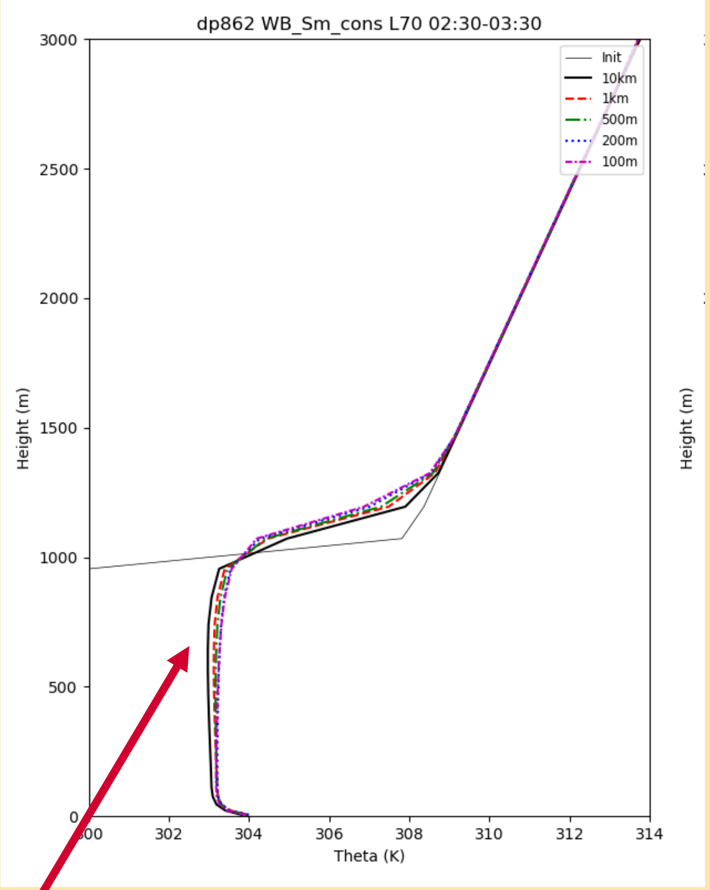
both lower resolution dependence and better near inversion!

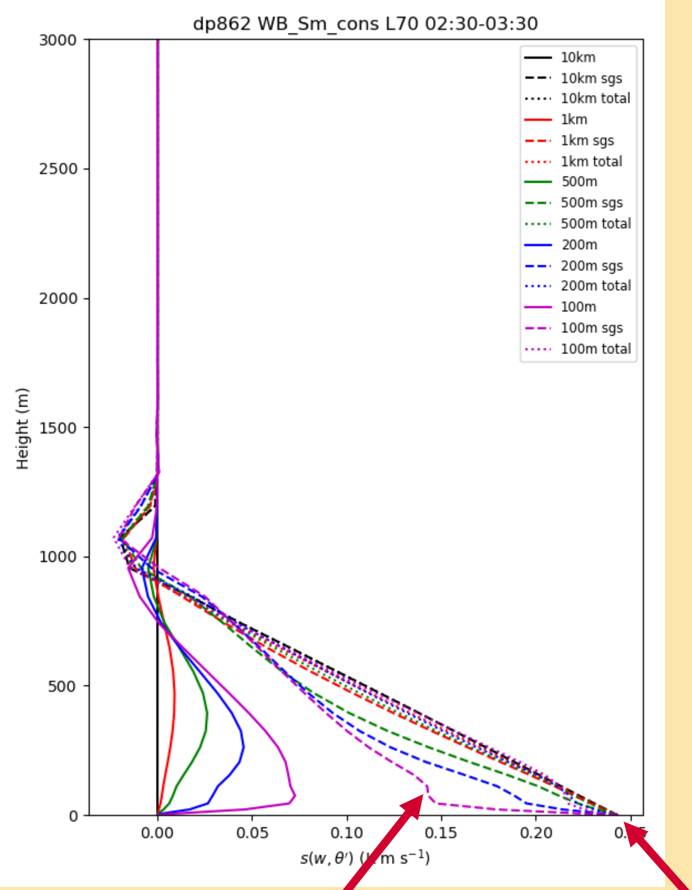
UM not locally conserving, struggles with explicit fluxes near the surface

Resolved scale trying to improve; sub-grid under-predict

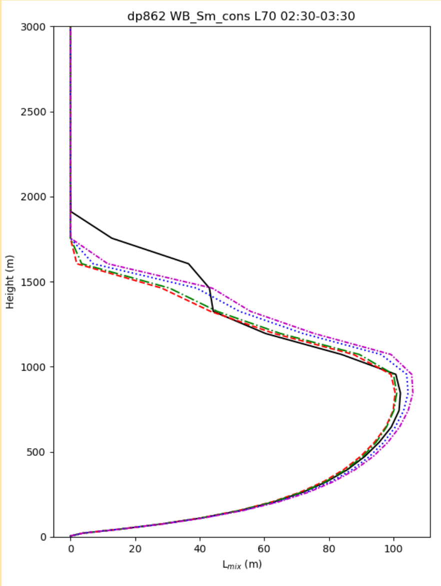
Improve both resolved scale and sub-grid, total better

Nakanishi-Niino Mellor-Yamada 1D BL L3





'UKV' 70-level set



$$\frac{1}{L_{mix}} = \frac{1}{\kappa(z+z_0)} + \frac{1}{L_{turb}} + \frac{1}{L_{stable}}$$

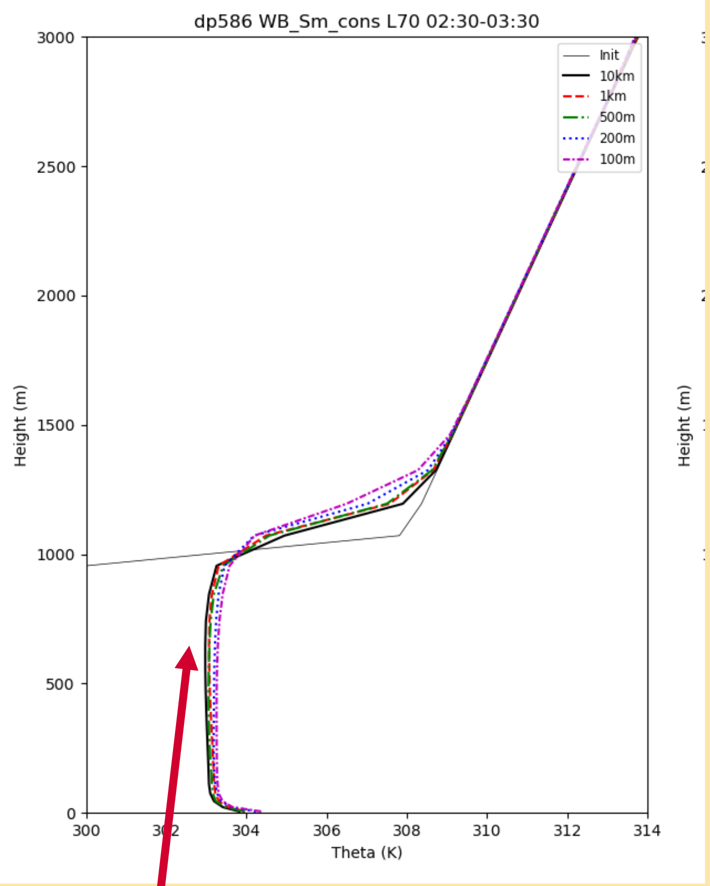
"designed to be controlled by the smallest length scale among the three length scales"

Close to 'Boundary-layer solution' at all resolutions.

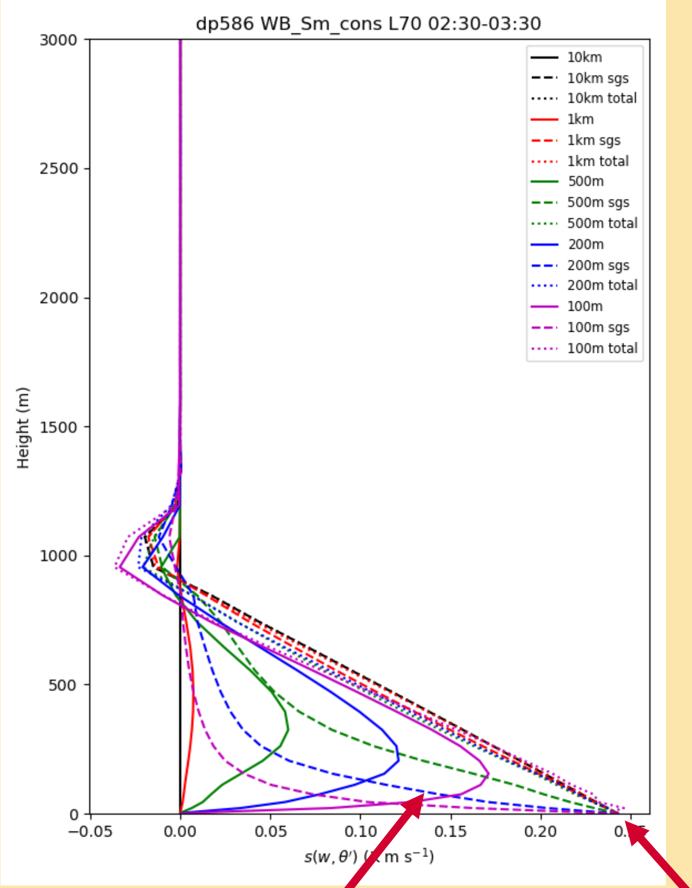
Mainly sub-grid flux even at 100 m.

Correct surface flux . Good mean flux profile.

3DTE Mk 1 L3

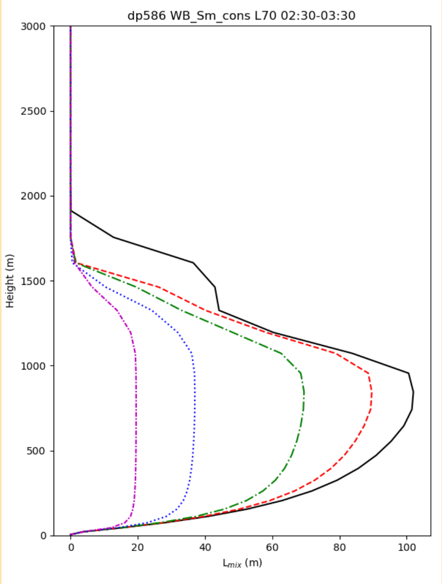


Fairly close to 'Boundary-layer solution' at all resolutions.
(A little too deep.)



Plausible 'hand-over' of flux from SGS to resolved.

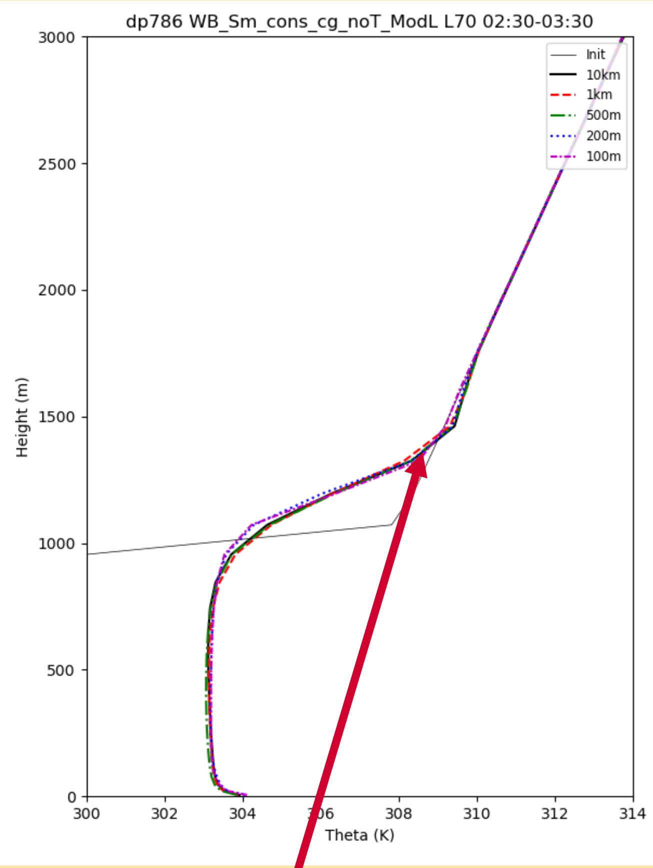
'UKV' 70-level set

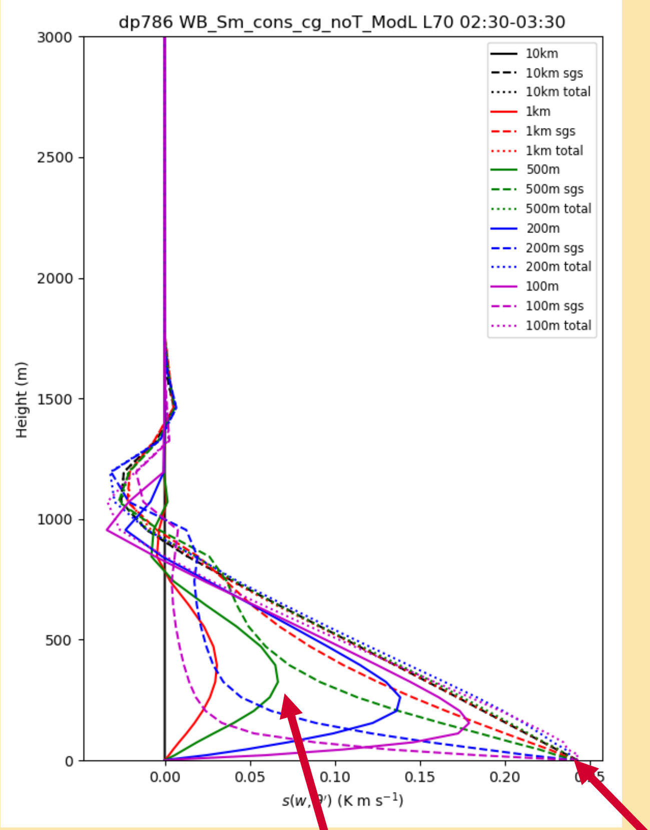


$$\frac{1}{L^2} = \frac{1}{L_{mix}^2} + \frac{1}{(C_S\Delta)^2}$$

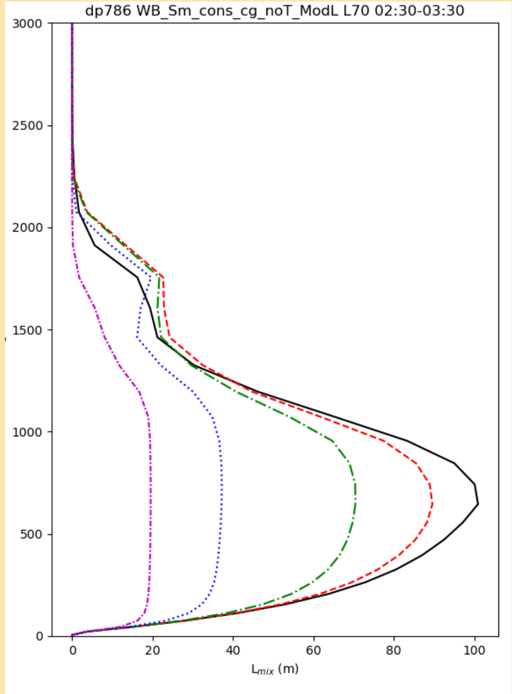
Correct surface flux .
Good mean flux profile.

3DTE Mk 2 (No Tilting Term) L3





'UKV' 70-level set



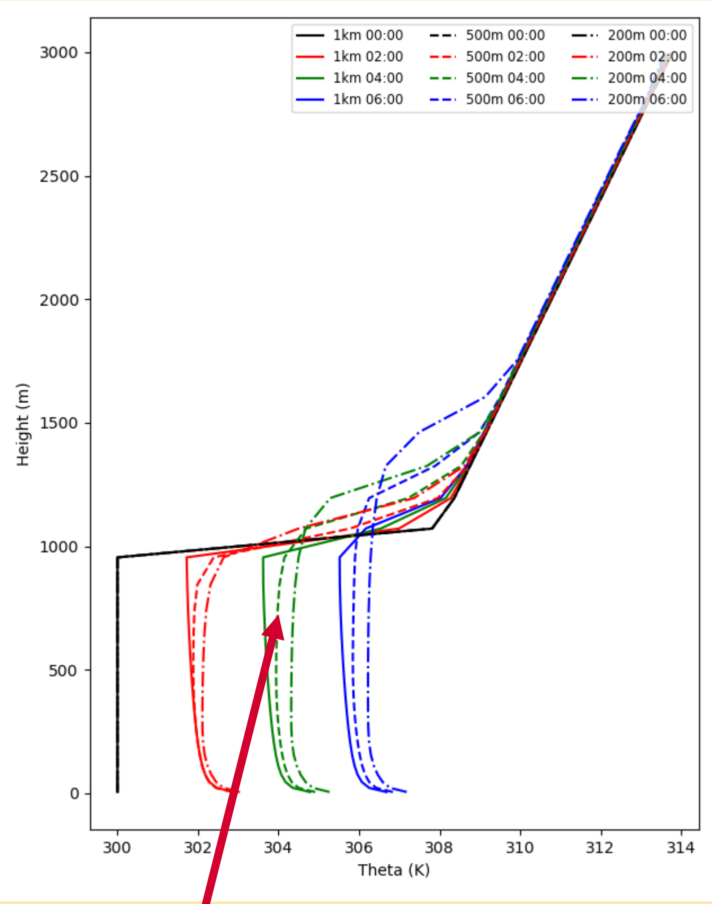
$$\frac{1}{L^2} = \frac{1}{L_{mix}^2} + \frac{1}{(C_S\Delta)^2}$$

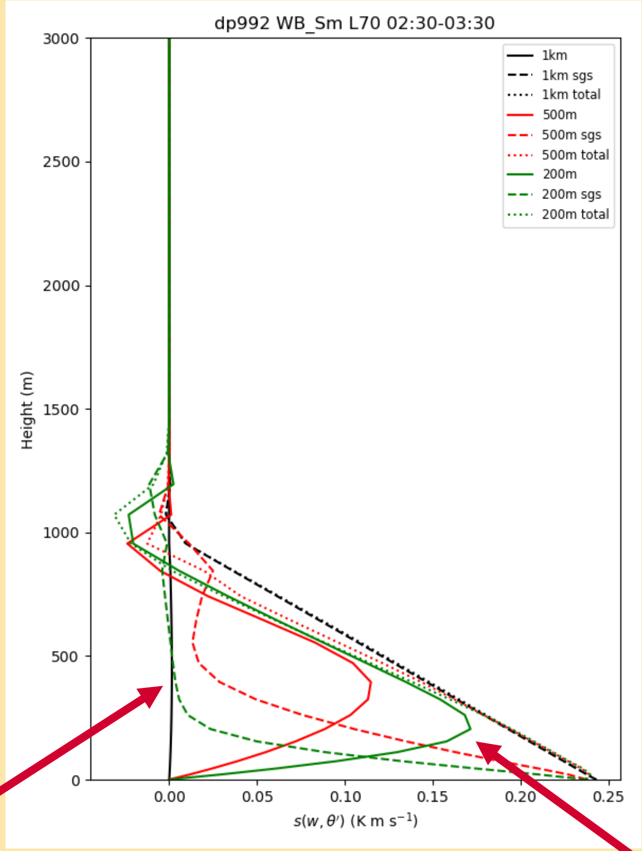
Fairly scale-independent but too deep.

Plausible 'hand-over' of flux from SGS to resolved.

Close to correct surface flux .
Good mean flux profile.

3DTE Mk 2 (No Tilting or CG) L2





scale-dependent

Different from Smag, suppressing all turbulence at 1km resolution.

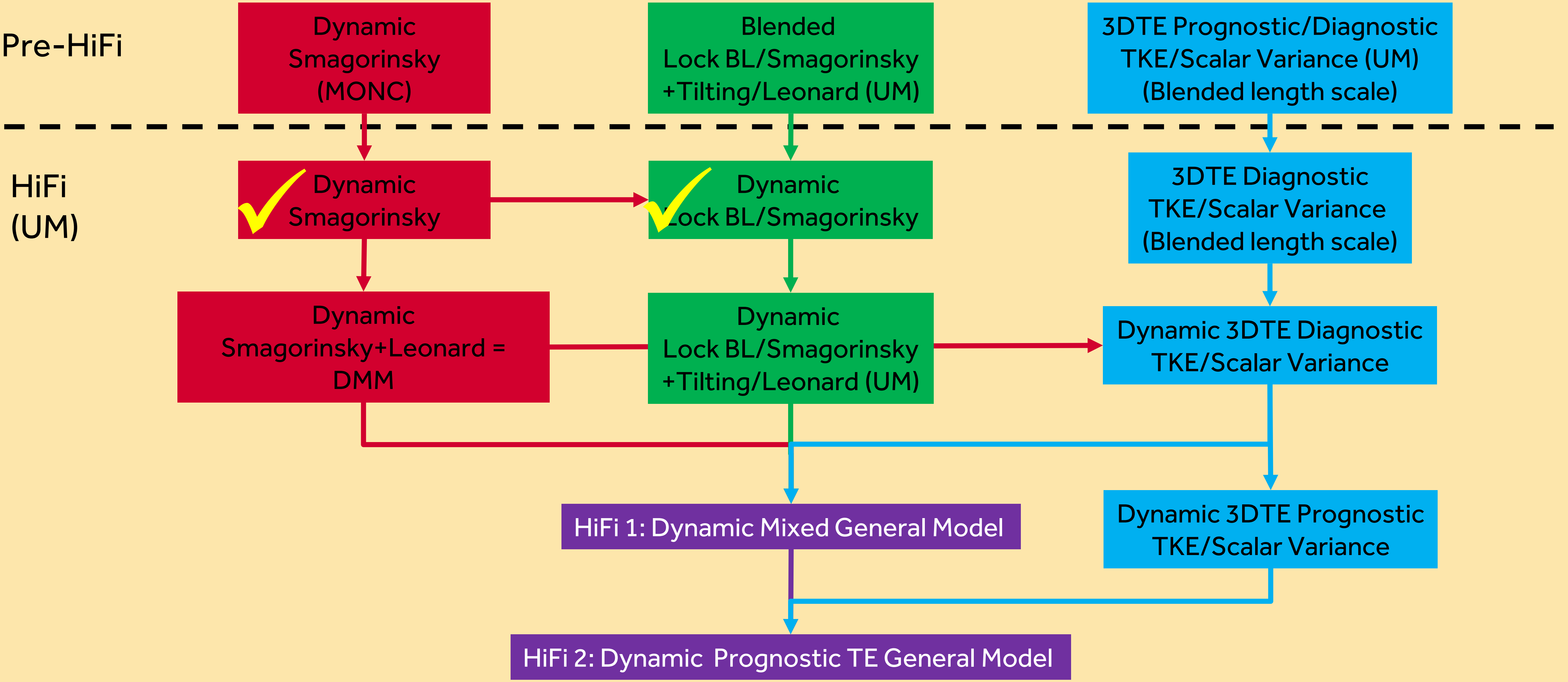
No overshoot of resolved scale as Smag, since $f_m(Ri = 0) > 1$ more SGS mixing

Conclusion

- **We need to go 3D schemes**: 1D schemes are only valid when ALL of the flux is sub grid. Not valid for grey zone. More to the point, 1D NNMY basically removes the resolved turbulence that should be there at high resolution
- **We need counter-gradient fluxes**: we need counter-gradient fluxes because in reality fluxes in top half of BL are counter gradient. We also need turbulence in the very stable inversion layer. Very hard to specify the length-scale here a priori.
- **We need dynamic length scale**: to work out better length scale specification especially near inversion and into deep clouds. Mk 1 has more consistent formulation of the cg term, but still very sensitive to the length scale.
- We may also need to add tilting terms for anisotropic production of turbulent fluxes (still in process). These are small in the CBL but we have shown they are important for deep clouds. We have already established benefits of tilting/Leonard terms for deep clouds but need to include them in the dynamic method if we are using it

Any Questions?

HiFi


Pre-HiFi
Dynamic Smagorinsky (MONC)
Blended Lock BL/Smagorinsky +Tilting/Leonard (UM)
3DTE Prognostic/Diagnostic TKE/Scalar Variance (UM) (Blended length scale)
HiFi (UM)
Dynamic Smagorinsky
Dynamic Lock BL/Smagorinsky
3DTE Diagnostic TKE/Scalar Variance (Blended length scale)
Dynamic Smagorinsky+Leonard = DMM
Dynamic Lock BL/Smagorinsky +Tilting/Leonard (UM)
Dynamic 3DTE Diagnostic TKE/Scalar Variance
HiFi 1: Dynamic Mixed General Model
Dynamic 3DTE Prognostic TKE/Scalar Variance
HiFi 2: Dynamic Prognostic TE General Model

Circle-A 3DTE:
The Full (approximate) Level 3 Solution

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Full (closed but un-approximated) prognostic equations for:
TKE (or $u_t^2 = s(u_k, u_k)$)
$s(\theta_L, \theta_L), s(q_t, q_t)$ and $s(\theta_L, q_t)$ from which we obtain $s(\phi, b)$ for any scalar.
Solve simultaneous equations for stress and scalar fluxes.

(Approximate) solution for 3D scalar fluxes:

Terms like $\lambda^2 \frac{\partial w^r}{\partial x} \frac{\partial \phi^r}{\partial x}$ in $s(w, \phi)$

$$s(u_i, \phi) = -\lambda u_t \left[S_H \frac{\partial \phi^r}{\partial x_i} + \Gamma_\phi \delta_{i3} \right] + S_H' \lambda^2 \left[S_M S_{ik}^r \frac{\partial \phi^r}{\partial x_k} + S_H'' \frac{\partial u_i^r}{\partial x_k} \left(S_H \frac{\partial \phi^r}{\partial x_k} + \Gamma_\phi \delta_{k3} \right) \right]$$

(3D) turbulent flux of ϕ

Down-gradient

Non-local (cg) (vertical only)

Shear Production/Tilting

(Einstein summation)

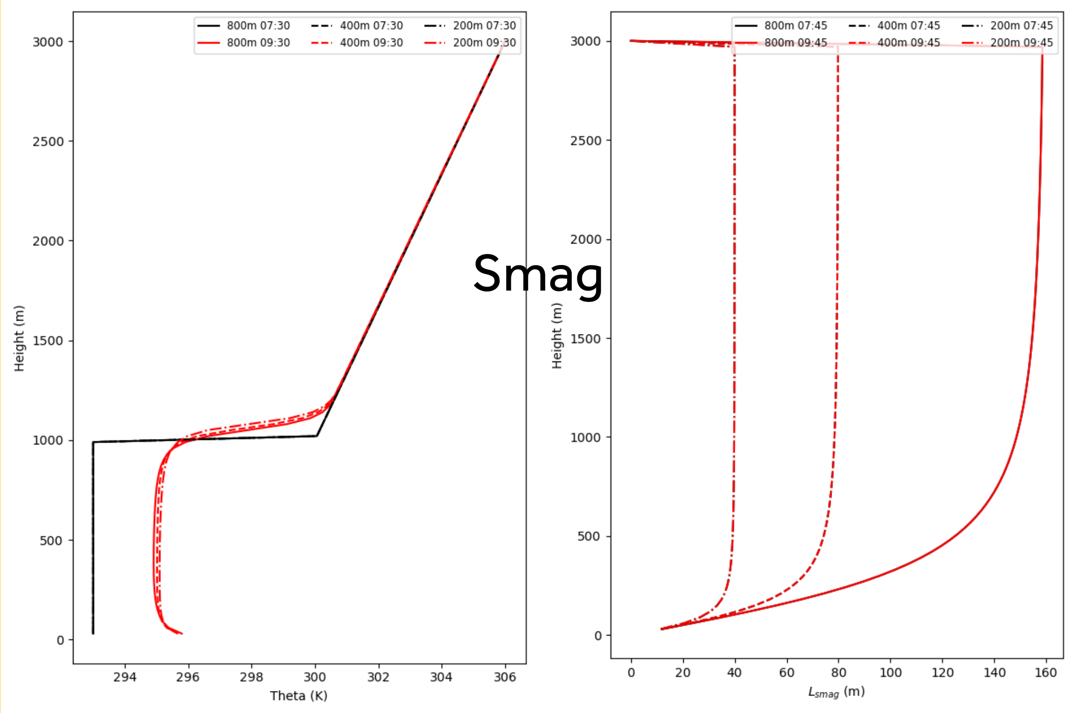
$$u_t^2 = 2e = s(u_k, u_k) \qquad S_{ij}^r = \left(\frac{\partial u_i^r}{\partial x_j} + \frac{\partial u_j^r}{\partial x_i} \right) \qquad \Gamma_\phi = -C_\phi \frac{s(\phi, b)}{u_t^2}$$

3DTE Mk 2

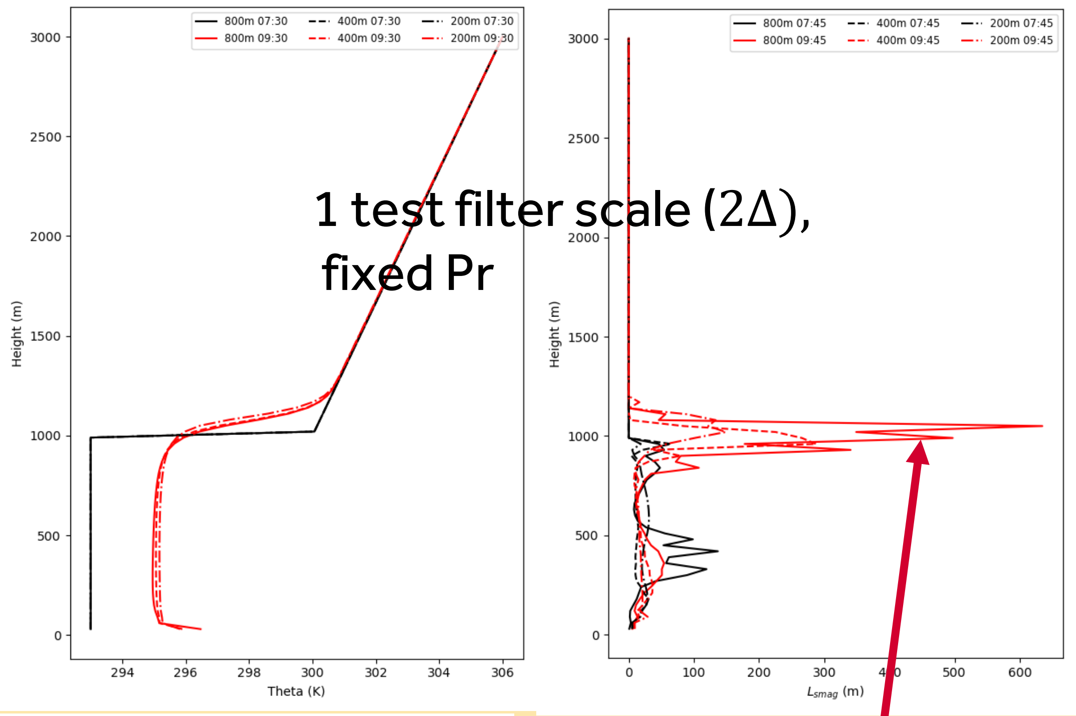
- Rewritten core code with separate down-gradient, counter-gradient and tilting terms
  - Still blended length scale – standard NNMY BL scale blended with Smagorinsky $C_s\Delta$.
  - Fully 3D tilting/Leonard flux.

- Removes Mk 1 inconsistencies.

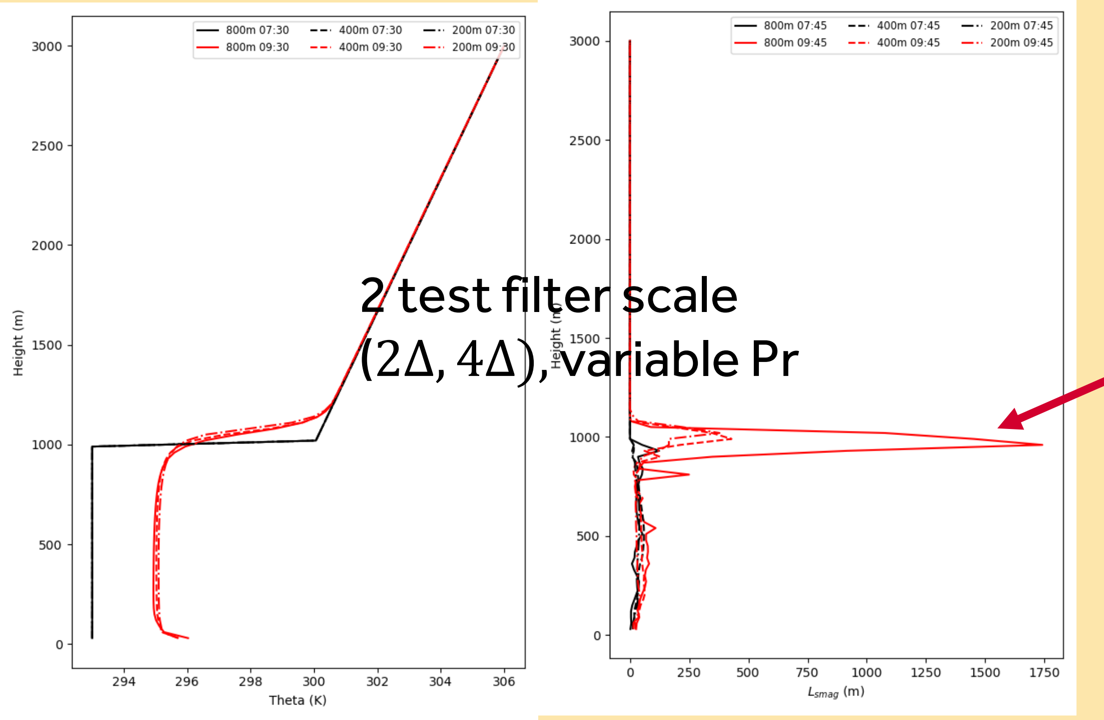
- Recommeded for use, but still in development.



Smag



1 test filter scale (2Δ), fixed Pr



2 test filter scale $(2\Delta, 4\Delta)$, variable Pr

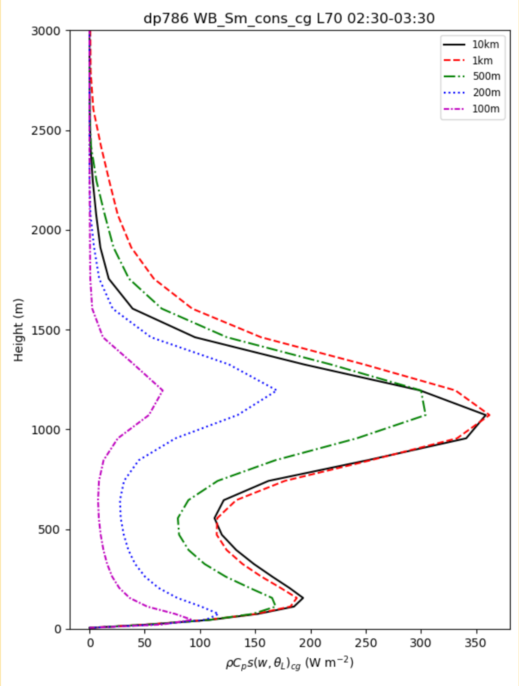
both provide a very large L_{smag} near inversion; change with growing turbulence

3DTE Mk 2 – length scale feedback

$$\frac{1}{L^2} = \left(\frac{1}{\kappa(z+z_0)} + \frac{1}{L_{turb}} + \frac{1}{L_{stable}}\right)^2 + \frac{1}{(C_s\Delta)^2}$$

NNMY L_{turb} is given by $L_{turb} = \frac{\int_0^{z_{top}} z\,e\,dz}{\int_0^{z_{top}} e dz}$,

i.e. depth of TKE layer.



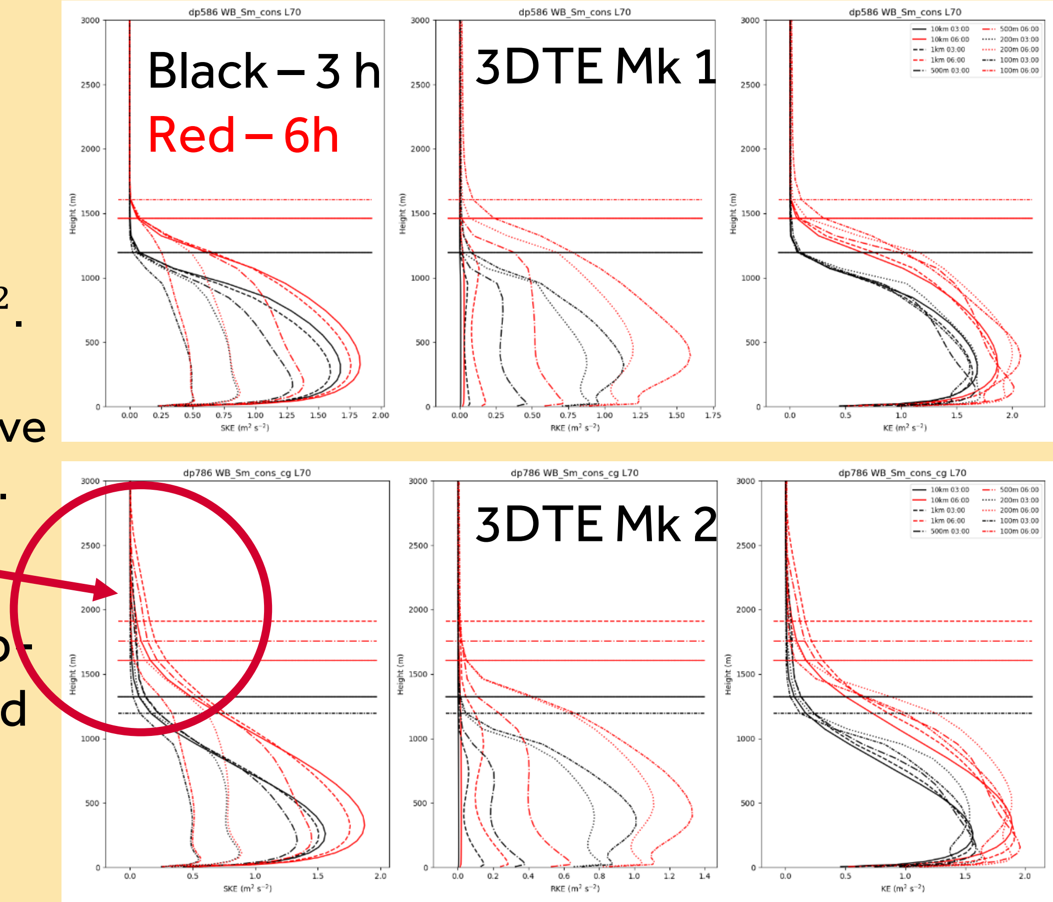
Counter-gradient flux

CG production term ∝ $L^2|\nabla\theta|^2$.

CG term too active in and above inversion – produces larger L^2.

Note: similar issue with sub-grid cloud when determined by scalar variance.

SG TKE Resolved TKE Total TKE



We need a better specification of turbulence length scale in the inversion layer!

3DTE Mk 1 vs Mk 2

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Mk 1:

- Modifications to NNMY 1D BL scheme to approximate 3DTE solution:
  - 3D shear in TKE production and hence Richardson number.
  - 3D Viscosity/diffusivity.
  - Blended length scale – standard NNMY BL scale blended with Smagorinsky $C_s\Delta$.
  - Vertical tilting/Leonard flux calculated using Kirsty Hanley's 1D code with local coefficient from 3DTE solution and blended length-scale.
- Already in UM release,
  - but note that **slight inconsistency** as some of counter-gradient and tilting terms are subsumed into diffusivity and viscosity (and hence used in horizontal), plus horizontal tilting terms are absent.

Mk 2:

- Rewritten core code with separate down-gradient, counter-gradient and tilting terms
  - **Fully 3D tilting/Leonard flux**.
- **Removes Mk 1 inconsistencies**.
- Still in development.

Circle-A 3DTE: The Full (approximate) Level 3 Solution

(Einstein summation)

$$s(u_i, u_j) = \frac{u_t^2}{3}\delta_{ij} - S_M \lambda u_t S_{ij}^r$$

Down-gradient

(3D) turbulent stress

$$+S'_M \lambda^2 \left(S_{jk}^r \frac{\partial u_i^r}{\partial x_k} + S_{ik}^r \frac{\partial u_j^r}{\partial x_k} - \frac{2}{3} S_{lk}^r \frac{\partial u_l^r}{\partial x_k} \delta_{ij} \right)$$

Shear Production/Tilting

$$-S''_M \lambda^2 \left[\left(S_H \frac{\partial b^r}{\partial x_i} + \Gamma_b \delta_{i3} \right) \delta_{j3} + \left(S_H \frac{\partial b^r}{\partial x_j} + \Gamma_b \delta_{j3} \right) \delta_{i3} - \frac{2}{3} \left(S_H \frac{\partial b^r}{\partial z} + \Gamma_b \right) \delta_{ij} \right]$$

Buoyant Production

$$u_t^2 = 2e = s(u_i, u_i)$$

$$S_{ij}^r = \left(\frac{\partial u_i^r}{\partial x_j} + \frac{\partial u_j^r}{\partial x_i} \right)$$

$$\Gamma_\phi = -C_\phi \frac{s(\phi, b)}{u_t^2}$$

Lengthscale blending

- Blackadar blending (away from surface): $\frac{1}{L} = \frac{1}{C_s\Delta} + \frac{1}{L_{BL}}$ or $\frac{1}{L^2} = \frac{1}{(C_s\Delta)^2} + \frac{1}{L_{BL}^2}$